REVISITING THE EXCHANGE-RATE PASS-THROUGH: A GENERAL EQUILIBRIUM PERSPECTIVE

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Abstract

A large literature estimates the exchange-rate pass-through to prices (ERPT) using reduced-form approaches. The results from these studies are an important input at Central Banks. We study the usefulness of these empirical measures for actual monetary policy analysis and decision making, emphasizing two main shortcomings that arise naturally from a general equilibrium perspective. First, while the empirical literature computes a single ERPT measure, in a general equilibrium model the evolution of the exchange rate and prices will differ depending on the shock hitting the economy. Accordingly, we distinguish between conditional and unconditional ERPT measures, showing that they can lead to very different interpretations. Second, in a general equilibrium model the ERPT crucially depends on the expected behavior of monetary policy; although the empirical literature cannot account for this. As a result, results from this literature might provide a misleading guide for monetary policy. We highlight the quantitative relevance of these with a DSGE model of a small and open economy with sectoral distinctions, real and nominal rigidities, and a variety of driving forces; estimated using Chilean data. *


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1 Introduction

The exchange-rate pass-through (ERPT) is a measure of the change in the price of a good (or basket of goods) after a change in the nominal exchange rate (NER), computed at different horizons after the initial movement in the NER. There is a large literature estimating the ERPT using a reduced-form empirical approach,\(^1\) such as vector auto-regressions (VAR) or single-equation models.\(^2\) Recently this topic has received a renewed interest, since many countries experienced large depreciations after the Tapering announcements by the FED in 2013.

The estimates of the ERPT coefficient are not only an important part of the international macroeconomics literature: they are quite relevant for actual monetary policy as well. This can be argued from three different perspectives. First, in the vast majority of Central Banks one can find studies estimating the ERPT for the particular country. Second, international institutions such as IMF, BIS, and IADB, among others, also actively participate in this discussion. For instance, some of the flagship reports of these institutions (such as the World Economic Outlook by the IMF or the Macroeconomic Report by the IADB) include estimates of the ERPT and use them to draw policy recommendations. Moreover, a significant number of papers in this literature comes from economist working at these institutions. Finally, it is easy to find references to the ERPT in many Monetary Policy Reports, proceedings from policy meetings, and speeches by board members at many Central Banks.

In this way, when domestic currencies experience large movements in nominal terms (depreciations in particular) Central Banks and other policy related institutions use the available estimates of the ERPT for two purposes. One is to try to predict the effect that the movement in the NER will have on inflation. The other is to assess ex-post why the behavior of inflation after a particular NER movement has been different to past experiences (for instances, arguing that the ERPT has changed over time, trying to provide a rationale for that change). In light to this widespread use, in this paper we question the usefulness of these empirical ERPT measures for monetary policy analysis and decision making, highlighting two relevant shortcomings that arise naturally from the perspective of general equilibrium models.

The first is related with the endogeneity of the NER and the sources behind its fluctuations. In the empirical literature, the ERPT is computed by trying to isolate exogenous movements in the NER, tracking then how different price measures react to these movements.\(^3\) In contrast, in a general equilibrium model the NER (as any other endogenous variable) can experience different dynamics depending on the particular shock that is hitting the economy, generating a co-movement between the NER and prices (and thus the ERPT) that can differ by to the source of the NER movement. Accordingly, we distinguish between conditional and unconditional or aggregate ERPT measures. The former refers to the ratio of the percentage change in a price index relative to that in the NER that occurs conditional on a given shock, computed at different horizons after the shock. The latter is the analogous ratio obtained from the reduced-form methodologies.

In the context of an extremely simple linear and dynamic model we analytically show that the unconditional ERPT obtained using a VAR approach is a weighted average of the conditional ERPTs in the model. Thus, to the extent that the conditional ERPTs are significantly different depending\(^4\)
on the shock, the empirical measures will provide a biased assessment of the expected relationship between the NER and prices.\footnote{Moreover, in the case in which the average of conditional ERPTs is not equal to any particular conditional ERPT, using the unconditional ERPT will systematically miss the expected evolution of the NER and prices; some times in one direction, some times in the other, but never equal to the actual movement in these variables.}

In more complicated models, the mapping between unconditional and conditional ERPT cannot be obtained algebraically. Nonetheless, we propose two alternative measures of aggregate ERPT that can be computed for any particular model and that mimic what an econometrician from the empirical literature will obtain if the general equilibrium model was the true data generating process. This allow us to compare the unconditional and conditional ERPT measures to assess the likely difference between them. To establish the quantitative relevance of this distinction we estimate a fully-fledge dynamic and stochastic general equilibrium (DSGE) model with sectoral distinctions, nominal and real rigidities, driven by a wide variety of structural shocks. We follow a Bayesian approach with quarterly Chilean data from 2001 to 2016.\footnote{Chile is an interesting case of study for several reasons. First, is a large commodity exporter with with a high degree of financial capital mobility; which makes relatively easy to identify the sources of foreign shocks. Second, since 2001 the Central Bank has followed a flexible inflation targeting strategy, that has been stable during the sample and it is consider as one of the success cases of inflation targeting, particularly in Latin America. This greatly facilitates the estimation of a DSGE model, without having to deal with possible shifts in the monetary policy framework. Finally, the exchange rate has moved freely most of the time during this sample, which is quite useful to show how diverse shocks may affect the NER. Nonetheless, the main points made in the paper are conceptually quite general, going beyond the particular country chosen for the estimation.}

Using the estimated model we show that the ERPT conditional on the two main drivers of the NER (a common trend in international prices and shocks affecting the interest parity condition) are quantitatively different, both in the short and in the long run, and they also differ depending on the measure of inflation (aggregate, tradable or non-tradable). At the same time, the unconditional ERPT lies between these two, and it is comparable with empirical estimates available in the literature for Chile. Overall, this evidence points to the importance of identifying the source of the shock that originates the NER change in discussing the likely effect on prices.

The second issue is related with monetary policy itself. In dynamic models the evolution of any endogenous variable (the NER and inflation in particular) crucially depends on the expected path of monetary policy. How this fundamental fact is captured in the empirical ERPT estimates is not clear. It might be argued that in these estimates it is implicitly assumed that monetary policy follows a policy rule that captures the “average” behavior followed by the central bank, during the sample analyzed. However, as there is no explicit description of this rule, it is hard to know what the central bank is assumed to be doing (and expected to do) in the estimated ERPT coefficient. Thus, to use these reduced-form estimates as a way to forecast the likely dynamics of inflation after a movement in the NER (as many do in policy related discussion) neglects the fact that monetary policy (both actual and expected) will influence the dynamics in the economy. If anything, what would be desirable to have is several ERPT measures, one for each alternative expected path for monetary policy that the Central Bank might consider. However, these cannot be computed using the methodologies applied in the empirical literature.

To quantify this problem we use our estimated DSGE model to explore how the ERPT (both conditional and unconditional) varies with different expected paths for monetary policy. In particular, we compare the benchmark ERPT, that assumes that policy behaves according to an estimated Taylor-type rule, with alternatives in which the central bank announces that, for a given number of periods, it will maintain the policy rate in the level that existed before the shock, returning to the estimated rule
afterwards. In principle, it is not clear how the ERPT will differ in these alternative situations: while a more dovish policy will produce a higher inflation, it will also induce a further nominal depreciation; thus it is not clear how the ERPT ratio might evolve. The results using the estimated model indicate that the conditional ERPT is significantly altered by different expected policy paths for some shocks but not for others. Additionally, the unconditional ERPT is also different for alternative interest rate paths. In sum, using the estimated ERPT provides an incomplete (and maybe misleading) assessment of the alternative policy options, and the expected dynamics under each of them.

Some papers in the literature study some aspects related to these issues we raise. The papers by Shambaugh (2008) and Forbes et al. (2015) use VAR models with alternative identification assumptions to estimate how several shocks might generate different ERPTs; in the same spirit as our definition of conditional pass-through. We see our work as complementary to theirs from two perspectives. First, these studies do not show how these conditional ERPT measures compares with unconditional ones; a comparison that we explicitly perform to understand the bias that looking at the unconditional ERPT could generate. Second, the identified shocks in these papers are too general relative to what can be specified in a DSGE model, thus our approach can provide a more precise description of the relevant conditional ERPTs.

A study that does use an estimated DSGE model to compute conditional ERPTs is Bouakez and Rebei (2008), which estimates the model for Canada, providing also a measure that would qualify as unconditional ERPT. Our paper builds on these results by providing an unconditional ERPT measure that is directly comparable to the methodology implemented in the empirical literature, and by analyzing the specific relationship between the measures obtained in the reduced-form approaches with the dynamics implied by a DSGE model. Moreover, our estimated DSGE model has a richer sectoral structure, allowing to characterize not only the ERPT for total inflation, but also that for different prices such as tradables and non-tradables. Corsetti et al. (2008) also explore structural determinants of the ERPT from a DSGE perspective and assess possible biases in single-equation empirical methodologies. While our paper shares many common points with this study, we additionally provides a quantitative evaluation of these biases by using an estimated DSGE model. Still, none of these studies explore the second issue regarding expected monetary policy that we do analyze.

Another group of papers explore several relevant aspects of the relationship between monetary policy and ERPT. For instance, Taylor (2000), Gagnon and Ihrig (2004) and Devereux et al. (2004) use dynamic general equilibrium models to see how monetary policy can alter the ERPT, proposing that a greater focus on inflation stabilization can provide an explanation to why the empirical measures of ERPT seems to have declined over time in many countries. Others have analyzed how monetary policy should be different depending on structural characteristics associated with the ERPT, such as the way currency at which international prices are set, the degree of nominal rigidities, among others. Some examples are Devereux et al. (2006), Engel (2009), Devereux and Yetman (2010), and Corsetti et al. (2010). Our paper explores a related but more specific aspect of monetary policy: the role of

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6 This exercise tries to mimic what would happen if the policy maker is presented with an estimated ERPT coefficient that is relatively low and convinces itself that the likely effect on inflation will be small, deciding not to change its policy.

7 In the case of Shambaugh (2008) long-run restrictions are used, generating shocks such as relative demand, relative supply, nominal, among others. However, in the DSGE model there are a number of shocks that can be included in each of these categories, each of them generating different conditional ERPTs. In the case of Forbes et al. (2015), shocks are identified by sign restrictions. In our estimated DSGE model, as we mentioned, two shocks explain almost 80% of fluctuations in the NER; but these two generate the same sign responses for those variables that can be observed. Thus the VAR strategy with sign restrictions cannot distinguish them, although in our analysis they generate clearly different conditional ERPTs.
the expected policy path to determine the ERPT. As we have argued, given the widespread use of empirical ERPT measures at Central Banks and policy related institutions, this point is crucial in providing a useful input for policy makers.

The rest of the paper is organized as follows. Section 2 describes the empirical strategies used in the literature and their relationship with DSGE models. The quantitative DSGE model and the estimation strategy is described in Section 3. In Section 4 we compare the conditional and unconditional ERPTs obtained from the estimated model, while Section 5 analyzes the role of alternative monetary policy paths to determine the ERPT. Conclusions are presented in Section 6.

2 The Empirical Approach to ERPT and DSGE Models

In this section we first the describe two methodologies generally used in the reduced form literature to estimate the ERPT (single-equation and VAR models). We then use a general linearized DSGE model to introduce the concept of conditional ERPT. Finally, we discuss the relationship between the conditional ERPT from the DSGE model and the measured obtained using a VAR approach, both in a very simple setting and in a general model, leading to two measures of unconditional ERPT in the DSGE model that are comparable to those obtained with a VAR.

2.1 The Empirical Approach

The empirical literature generally features two alternative approaches to compute the ERPT: single-equation, distributed-lag models and vector auto-regressions. In the first the estimated model takes the form,

\[ \pi^j_t = \alpha + \sum_{j=0}^{K} \beta_j \pi^S_{t-j} + \gamma c_t + \nu_t, \]  

(1)

where \( \pi^j_t \) denotes the log-difference in the price of a good (or basket of goods) \( j \), \( \pi^S_t \) is the log-difference of the NER, \( c_t \) is a vector of controls (either external to the economy or domestic) and \( \nu_t \) is an error term. The parameters \( \alpha, \beta_j, \) and \( \gamma \) are generally estimated by OLS, and the ERPT \( h \) periods after the movement in the NER is computed as \( \sum_{j=0}^{h} \beta_j \), representing the percentage change in the price of good \( j \) generated by a 1% permanent change in the NER.

The VAR strategy specifies a model for the vector of stationary variables \( x_t \) that includes \( \pi^S_t, \pi^j_t \), and generally other control variables, both of domestic and foreign origin. The reduced-form VAR(p) model is,

\[ x_t = \Phi_1 x_{t-1} + \cdots + \Phi_p x_{t-p} + u_t, \]  

(2)

where \( \Phi_j \) for \( j = 1, \ldots, p \) are matrix to be estimated, and \( u_t \) is a vector of i.i.d. reduced-form shocks, with zero mean and variance-covariance matrix \( \Omega \). Associated with \( u_t \) we can define the “structural” disturbances \( w_t \) as,

\[ u_t = P w_t, \]  

(3)

where \( P \) satisfies \( \Omega = PP' \), assuming the variance of \( w_t \) equals the identity matrix. In the empirical ERPT literature \( P \) is assumed to be lower triangular, obtained from the Cholesky decomposition of
Ω, and the ERPT $h$ periods ahead is defined as:

$$ERPT^V(h) = \frac{CIRF^V_{\pi_j,\pi_S}(h)}{CIRF^V_{\pi_S,\pi_S}(h)},$$

(4)

where $CIRF^V_{k,i}(h)$ is the cumulative impulse-response of variable $k$, after a shock in the position associated with variable $i$, $h$ periods after the shock. In other words, the ERPT is the ratio of the cumulative percentage change in the price relative to that in the NER, originated by the shock associated with the NER in the Cholesky order.\(^8\)

While both approaches can be find in the literature, here we use the VAR as a benchmark for several reasons. First, in the most recent papers the VAR approach is generally preferred. Second, the ERPT obtained from (1) assumes that after the NER moves, it stays in that value forever. In contrast, the measure (4) allows for richer dynamics in the NER after the initial change. Third, the OLS estimates from (1) will likely be biased, as most of the variables generally included in the right-hand side are endogenous. The VAR attempts to solve this strategy by including lags of all variables, and by means of the identification strategy, as long as the Cholesky decomposition is correct.\(^9\) Finally, the VAR model might, in principle, be an appropriate representation of the true multivariate model (as we will discuss momentarily), but the same is generally not true for a single-equation model.

### 2.2 DSGE models and Conditional ERPT.

The linearized solution of a DSGE model takes the form,

$$y_t = F y_{t-1} + Q e_t,$$

(5)

where $y_t$ is a vector of variables in the model (exogenous and endogenous, predetermined or not), $e_t$ is a vector of i.i.d. structural shocks, with mean zero and variance equal to the identity matrix,\(^10\) and the matrices $F$, and $Q$ are non-algebraic functions of the deep parameters in the model.\(^11\)

Using the solution, the ERPT conditional to the shock $e^j_t$ for the price of good $j$ is defined as,

$$CERPT^M_{j}(h) = \frac{CIRF^M_{\pi_j,e^j}(h)}{CIRF^M_{\pi_S,e^j}(h)},$$

(6)

This is analogous to the definition of $ERPT^V(h)$ in (4), with the difference that the response is computed after the shock $e^j_t$, and we can compute one for each shock in the vector $e_t$.

\(^8\)In general, it is assumed that $\pi^j_S$ is ordered before $\pi^j_I$ in the vector $x_t$. In addition, if the vector $x_t$ contains foreign variables and the country is assumed to be small relative to the rest of the world, these variables are ordered first in $x_t$ and the matrices $\Phi_j$ are assumed to have a block of zeros to prevent feedback from domestic variables to foreign ones at any lag.

\(^9\)Of course, we will describe in the next subsection how that assumption will generally not hold if a DGSE model is the true data generating process. But at least the VAR methodology attempts to deal with the endogeneity issue, while the single-equation, OLS based approach does not.

\(^10\)In general, the number of shock in $e_t$ will be less or equal to the number of variables in $y_t$.

\(^11\)This solution can be obtained by several methods after linearizing the non-linear equilibrium conditions of the model, and can be implemented in different packages, such as Dynare.
2.3 The Relationship Between VAR- and DSGE-based ERPT

We want to explore the relationship between \( ERP_T^V(h) \) and \( CERP_T^M_i(h) \), in order to construct a measure of unconditional ERPT from the DSGE model that is comparable to \( ERP_T^V(h) \). Relevant for this discussion is the work of Ravenna (2007), who explores conditions under which the dynamics of a subset of variables in the DSGE model can be represented with a finite-order VAR model. The general message is that is not obvious that a DSGE model will meet these requirements, implying that the relationship we wish to find can only be obtained analytically only for specific cases.

Consider then a hypothetical simple case in which the variables in the VAR are only \( x_t = [\pi^S_t, \pi^j_t]' \), where the DSGE features only two shocks \( (e_t = [e^1_t, e^2_t]') \), and that the VAR representation of the DSGE coincides with equation (5), i.e.

\[
x_t = F x_{t-1} + Q e_t,
\]

In this simple case, someone estimating a VAR(1) on the vector will obtain \( \Phi_1 = F \) and \( \Omega = QQ' \). In most DSGE models the matrix \( Q \) is not generally lower triangular, so \( P \neq Q \), however we assume the econometrician uses a Cholesky decomposition as described in the previous subsection and computes \( ERP_T^V(h) \) as in (4) using the responses to the first shock.

Under these assumptions Appendix A.1 shows that

\[
ERP_T^V(h) = CERP_T^M_1(h)\omega_1(h) + CERP_T^M_2(h)\omega_2(h).
\] (7)

In other words, the ERPT obtained from the VAR will be a weighted sum of the conditional ERPTs in the DSGE model. For \( h = 0 \) the weight \( \omega_i(0) \) corresponds to the fraction of the forecast-error variance of the NER, at horizon \( h = 0 \), explained by the shock. For \( h > 0 \) the weight \( \omega_i(k) \) is equal to \( \omega_i(0) \) adjusted by how different the response of the NER in horizon \( h \) is relative to the moment the shock \( i \) hits the economy (\( h = 0 \)). In simpler terms, the ERPT obtained from the VAR is weighted sum of those conditional ERPT in the DSGE model, where the weights depend on how import each shock is in explaining the fluctuations in the NER.

For general models, as the VAR representation of the DSGE is not guaranteed to be finite and the DSGE might feature more shocks than the variables used in the VAR, we propose two alternatives to compute the unconditional ERPT in the DSGE model. The first is inspired by the simple \( 2 \times 2 \) case. Let \( k \) be the position of the inflation of interest in the vector \( y_t \), and let \( s \) be the position of the nominal depreciation rate in the vector \( y_t \). Then we define,

\[
UERP_T^M(k)(h) \equiv \sum_{i=1}^{n_e} CERP_{k,i}^M(h)\omega_i(h),
\] (8)

where \( n_e \) is the number of shock in the vector \( e_t \), \( CERP_{k,i}^M(h) \) is computed as in (6), and \( \omega_i(h) \) is analogous to the one in (7) (see Appendix A.1 for details).

The second measure of unconditional ERPT answers the following question: what would be the ERPT that someone using the empirical VAR approach would estimate if she has an infinite sample of

\footnote{A related issue is analyzed by Fernández-Villaverde et al. (2007), showing conditions under which the shocks identified in a VAR for a subset of the variables in the DSGE can capture the same shocks as those in the DSGE model. However, as the empirical VAR literature of ERPT does not claim that is identifying any particular shock that can be interpreted from a DSGE model, this aspect is not as relevant for our discussion.}
the variables commonly used in that literature, generated by the DSGE model? We call this alternative unconditional ERPT using a Population VAR, labeled as $U_{ERPT}^{PV}(h)$; which is analogous to (4) but when the matrices $\Phi_j$ and $\Omega$ are obtained from the population (i.e. unconditional) moments computed from the solution of the DSGE model.\footnote{Appendix A.2 details how this is computed.}

Therefore, for any particular DSGE model, we have two alternatives unconditional ERPTs to compare them with the conditional ones, in order to assess the difference between them. In the next section we introduce a DSGE model, estimated with Chilean data, to provide a quantitative evaluation of these differences.

3 The Quantitative DSGE Model

In this section we first describe the different parts of the DSGE model, leaving the optimality and equilibrium conditions to Appendix B. We then discuss the parametrization strategy. Finally, we describe the main driving forces for NER movements according to the estimated model, and provide intuition on how these shocks propagate to the economy.

3.1 Overview

The model is relatively large, for our goal is to provide a satisfactorily account of the dynamics in the data and, in this way, generate an appropriate quantitative evaluation of the issues we raise. Our setup is one of a small open economy model with both nominal and real rigidities, and incomplete international financial markets. There are three goods produced domestically: Commodities ($Co$), Non-tradables ($N$), and an exportable good ($X$). The first is assumed to be an exogenous endowment that is fully exported, while the other two are produced by combining labor, capital, imported goods ($M$, which are sold domestically through import agents) and Energy ($E$). Consumption (both private and public) and investment goods are a combination of $N$, $X$ and $M$ goods.\footnote{Final consumption also requires Energy and Food, which are the items that are considered in the non-core part of inflation in Chile. These are assumed to be produced by combining $X$ and $M$ goods; although having a different price dynamic in the short run.} Households derive utility from consumption and leisure, borrow in both domestic- and foreign-currency-denominated bonds, and have monopoly power in supplying labor. Moreover, we assume imperfect labor mobility across sectors. Household’s utility exhibits habits in consumption, and investment is subject to convex adjustment costs.

Firms in the $X$, $N$ and $M$ sectors are assumed to have price setting power, through a monopolistic-competition setup. The problem of choosing prices, as well as that of setting wages, is subject to Calvo-style frictions, with indexation to past inflation. The possibility of indexation to aggregate inflation is relevant to determine the ERPT to different goods, particularly non-tradables.

Monetary policy sets the interest rate on domestic bonds, following a Taylor-type rule. Fiscal policy is assumed to finance an exogenous stream of consumption using lump-sum taxes and proceeds from the ownership of part of commodity production. The model is completed by the rest of the world, where international prices and interest rates are set exogenously, following the small-open economy assumption.
3.2 Households

There is a representative household that consumes, works, saves, invests and rents capital to the producing sectors. Her goal is to maximize,

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left\{ \frac{(C_t - \phi_C \tilde{C}_{t-1})^{1-\sigma}}{1-\sigma} - \kappa_t \left( \xi_{t}^{h_{X}^{j+\varphi}} + \xi_{t}^{h_{N}^{j+\varphi}} \right) \right\}$$

where $C_t$ is consumption and $h_{t}^{j}$ for $J = X, N$ are hours worked in sector $J$. $\tilde{C}_{t}$ denotes aggregate consumption (i.e. the utility exhibits external habits), $\kappa_t \equiv (\tilde{C}_{t} - \phi_C \tilde{C}_{t-1})^{-\sigma}$. $\xi_t^{h_{j}}$ and $\xi_t^{h_{j}}$ are preference shocks: the former affects inter-temporal decisions, while the latter is a labor supply shifter in sector $J = X, N$.

The budget constraint is

$$P_t C_t + S_t B_t^* + B_t + P_t I_t^N + P_t I_t^X = h_{t}^{X,d} \int_{0}^{1} W_t^{X} (j) \left( \frac{W_t^{X} (j)}{W_t^{X}} \right)^{-\epsilon_W} dj +$$

$$h_{t}^{N,d} \int_{0}^{1} W_t^{N} (j) \left( \frac{W_t^{N} (j)}{W_t^{N}} \right)^{-\epsilon_W} dj + S_t R_t - B_t + R_t - 1 B_t - 1 + P_t^N R_t^N K_t^N +$$

$$P_t^X R_t^X K_t^X + T_t + \Pi_t.$$  

Here $P_t$ the price of the consumption good, $S_t$ the exchange rate, $B_t^*$ the amount of external bonds bought by the household in period $t$, $B_t$ amount of local bonds bought by the household in $t$, $P_t^I$ is the price of the investment good, $I_t^J$ investment in capital of the sector $j$, $h_{t}^{J,d}$ is labor demand in sector $j$, $R_t$ is the external interest rate, $R_t^d$ is the real rate from renting their capital to firms in sector $J$, $P_t^J$ is the price of goods $J$, $T_t$ are transfers made by the government and finally $\Pi_t$ has all the profits of the firms in all sectors.

The formulation of the wage-setting problem follows Schmitt-Grohe and Uribe (2006). In this setup, households supply a homogeneous labor input that is transformed by monopolistically competitive labor unions into a differentiated labor input. The union takes aggregate variable as given and decides the nominal wage, while supplying enough labor to meet the demand in each market. The wage of each differentiated labor input is chosen optimally each period with a constant probability $1 - \theta_{W,J}$ for $J = \{X, N\}$. When wages cannot be freely chosen they are updated by $(\tilde{\pi}_{t-1})^{\tilde{\gamma}_{W,J}} \tilde{\pi}^{1-\tilde{\gamma}_{W,J}}$, with $\tilde{\pi}_{t-1}$ denoting previous-period CPI inflation and $\tilde{\pi}$ the inflation target set by the Central Bank.

3.2.1 Consumption Goods

Consumption $C_t$ is composed by three elements: core consumption ($C_t^{NFE}$), food ($C_t^{F}$) and energy ($C_t^{E}$). For simplicity, food and energy consumption are assumed exogenous and normalized to one (so total and core consumption are equal). In contrast the price of the consumption good will be a composite of the price of the core good, energy and food the following way:

$$P_t = (P_t^{NFE})^{1-\gamma_{FC}} (P_t^{FC})^{\gamma_{FC}} (P_t^{EC})^{\gamma_{EC}}$$

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15 In equilibrium $\tilde{C}_t = C_t$.

16 This utility specification follows Gali et al. (2012), and it is designed to eliminate the wealth effect on the supply of labor while keeping separability between consumption and labor.
Where $P_{t}^{NFE}$ is the price of core consumption, $P_{t}^{F}$ is the price of food and $P_{t}^{E}$ is the price of energy.\footnote{17} We further assume that the price of both $F$ and $E$ relative of that for the tradable composite ($T$, defined below) follows an exogenous process ($p_{t}^{F}$ and $p_{t}^{E}$ respectively).\footnote{18}

Core consumption is a composite of non-tradable consumption $C_{t}^{N}$ and tradable consumption $C_{t}^{T}$, while the latter is composed by exportable $C_{t}^{X}$ and importable $C_{t}^{M}$ goods,

\[
C_{t}^{NFE} = \left[ \gamma^{1/\varepsilon}(C_{t}^{N})^{\varepsilon-1/\varepsilon} + (1 - \gamma)^{1/\varepsilon}(C_{t}^{T})^{\varepsilon-1/\varepsilon} \right]^{\varepsilon-1/\varepsilon}
\]

\[
C_{t}^{T} = \left[ \gamma^{1/\varepsilon_{T}}(C_{t}^{X})^{\varepsilon_{T}-1/\varepsilon_{T}} + (1 - \gamma^{T})^{1/\varepsilon_{T}}(C_{t}^{M})^{\varepsilon_{T}-1/\varepsilon_{T}} \right]^{\varepsilon_{T}-1/\varepsilon_{T}}
\]

\[
C_{t}^{J} = \int_{0}^{1} G(C_{t}^{J}(i), \xi_{t}^{J})di
\]

The last equation specifies that exportable, importable and non-tradable consumption are made of a continuum of differentiated goods in each sector, combined by an aggregator $G$, which we assume it features a constant elasticity of substitution $\varepsilon_{J} > 1$ for $J = \{X, M, N\}$. Moreover, it is assumed that the aggregator is subject to exogenous disturbances ($\xi_{t}^{J}$), generating markup-style shocks in the pricing decisions by firms as in Smets and Wouters (2007).

### 3.2.2 Capital and Investment Goods

The evolution of the capital stock in sector $J$ is

\[
K_{t}^{J} = \left[ 1 - \Gamma \left( \frac{I_{t}^{J}}{I_{t-1}^{J}} \right) \right] u_{t}I_{t}^{J} + (1 - \delta)K_{t-1}^{J},
\]

for $J = \{X, N\}$. It is assumed that installed capital is sector-specific and there are adjustment costs to capital accumulation with $\Phi(\cdot) > 0$ and $\Phi''(\cdot) > 0$. $u_{t}$ is a shock to the marginal efficiency of investment.\footnote{19}

Households choose how much to invest in each type of capital, which constitutes the demand for investment. The supply of investment is assumed to be provided by competitive firms that have a technology similar to the consumption preferences of households, but with different weights and elasticities of substitution,

\[
I_{t} = \left[ \gamma^{1/\varepsilon_{t}}(\tilde{I}_{t}^{N})^{\varepsilon_{t}-1/\varepsilon_{t}} + (1 - \gamma)^{1/\varepsilon_{t}}(\tilde{I}_{t}^{T})^{\varepsilon_{t}-1/\varepsilon_{t}} \right]^{\varepsilon_{t}-1/\varepsilon_{t}}
\]

\[
\tilde{I}_{t}^{J} = \left[ \gamma_{T,I}^{1/\varepsilon_{T,I}}(\tilde{I}_{t}^{X})^{\varepsilon_{T,I}-1/\varepsilon_{T,I}} + (1 - \gamma_{T,I})^{1/\varepsilon_{T,I}}(\tilde{I}_{t}^{M})^{\varepsilon_{T,I}-1/\varepsilon_{T,I}} \right]^{\varepsilon_{T,I}-1/\varepsilon_{T,I}}
\]

Similar to consumption, each investment $\tilde{I}_{t}^{J}$ for $J = \{X, M, N\}$ is a continuum of the differentiated goods in each sector with the same elasticity of substitution as consumption, $\varepsilon_{J}$.
3.3 Firms

Besides Commodities (assumed to be an endowment), there are three sectors: exportable, importable and non-tradable. Firms in the importable sector buy an homogeneous good from foreigners and differentiate it, creating varieties which are demanded by households and firms. Firms in the exportable and non-tradable sector combine a value added created using labor and capital with a composite of the varieties sold by the importable sector to produce their final product.

Each firm in each sector supplies a differentiated product, generating monopolistic power. Given their marginal cost, they maximize prices a la Calvo with probability \( \theta_J \) for \( J = \{X, M, N\} \) of not being able to choose its price optimally each period. When they cannot choose its price, it is assumed to be updated according to: 

\[
\pi_t^J \equiv \left( (\pi_{t-1}^J)^{\theta_J} (\pi_{t-1})^{1-\theta_J} \right)^{\zeta_J} \pi_{t-1}^{1-\zeta_J}, \quad \text{with} \quad \pi_{t-1}^J \text{ being inflation of sector } J \text{ in the previous period.}
\]

In this way, the indexation specification is flexible enough to accommodate both dynamic as well as static (i.e. steady-state) indexation, with a backward-looking feedback that can be related to either sector specific or aggregate inflation; and we let the data tell the appropriate values for \( \theta_J \) and \( \zeta_J \) in each sector.

3.3.1 Sector \( M \)

Each firm \( i \) in this sector produces a differentiated product from an homogeneous foreign input with the technology \( Y_t^M(i) = M(i) \). The price of their input is given by \( P_{m,t} = S_t P_{M,t}^* \), where \( P_{m,t} \) is the price of the good that is imported in local currency and \( P_{M,t}^* \) is the price in foreign currency and is exogenously given.

3.3.2 Sector \( X \) and \( N \)

All firms in both sectors have the same format. Each firm \( i \) of sector \( J \) produces a differentiated product that is a combination of value added \( V_t^J(i) \) and an importable input \( M_t^J(i) \), which is a combination of a continuum of the goods sold by \( M \) sector and energy. They have the technology,

\[
Y_t^J(i) = (V_t^J(i))^{\gamma_J} (M_t^J(i))^{1-\gamma_J},
\]

where value added is produced by,

\[
V_t^J(i) = z_t^J \left[ K_{t-1}^J(i) \right]^{\alpha_J} \left[ A_t^J h_t^{J,d}(i) \right]^{1-\alpha_J}.
\]

\( z_t^J \) is a stationary technology shock, while \( A_t^J \) is a non-stationary stochastic trend in technology. To maintain a balance-growth path, we assume that both trends co-integrate in the long-run. In particular, we assume that \( \alpha_t = A_t^N / A_{t-1}^N \) is an exogenous process and \( A_t^X \) evolves according to,

\[
A_t^X = (A_{t-1}^X)^{1-\Gamma_X} (A_t^N)^{\Gamma_X}
\]

The factor demand for these firms can be solved in two stages:

1. Optimal production of \( V_t^J(i) \): Firms are price takers, so they choose the optimal combination of capital and labor to minimize their cost,

\[
\min_{K_{t-1}^J(i), h_t^J(i)} P_t^J R_t K_{t-1}^J(i) + W_t^J h_t^J(i) + \mu \left\{ V_t^J(i) - z_t^J \left[ K_{t-1}^J(i) \right]^{\alpha_J} \left[ A_t^J h_t^{J,d}(i) \right]^{1-\alpha_J} \right\}
\]
2. Optimal production of $Y^J_t(i)$: The cost minimization in this case is,

$$\min_{M^J_t(i), V^J_t(i)} MC^V_J V^J_t(i) + P^{ME}_t M^J_t(i) + \mu \{ Y^J_t(i) - [V^J_t(i)]^{\gamma_J} [M^J_t(i)]^{1-\gamma_J} \}$$

where $MC^V_J$ is the marginal cost of producing $V^J_t(i)$, which is the same for all firms, and $P^{ME}_t$ is the price of a composite between a continuum of the importable goods sold by the $M$ sector and energy; i.e.

$$P^{ME}_t = (P^M_t)^{1-\gamma_{EM}} (P^E_t)^{\gamma_{EM}}$$

As in the case of the household with Energy and Food, $M^J_t(i)$ can be interpreted as only the continuum of importable goods or the composite between energy and the importable goods, since firm take the quantity of energy as exogenous and so it has been normalized to one.

3.4 Commodity

The Commodity is assumed to be an exogenous and stochastic endowment, $Y^{Co}_t$ which has its own trend $A^{Co}_t$ that evolves that cointegrates with the other sectors as $A^{Co}_t = (A^{Co}_{t-1})^{1-\Gamma_{Co}} (A^N_t)^{\Gamma_{Co}}$. We assume $y^{Co}_t \equiv \frac{Y^{Co}_t}{A^{Co}_t}$ follows an exogenous process. The endowment is exported at the international price $P^{Cos}_t$. It is assumed that a fraction $\vartheta$ of commodity production is owned by the government and a fraction $(1-\vartheta)$ by foreigners.

3.5 Fiscal and Monetary Policy

The fiscal policy introduces an exogenous expenditure that is completely spent in non-tradable goods. The government receives part of the profits by the Commodity sector, can buy local bonds and gives transfers to the household $T_t$. Its budget constraint is

$$\vartheta S_t P^{Cos}_t Y^{Co}_t + R_{t-1} B^G_{t-1} = P^N_t G_t + T_t + B^G_t$$

Similarly to the household, government expenditure is composed by a composite of non-tradable varieties with elasticity of substitution $\epsilon_N$. We assume $g_t \equiv \frac{G_t}{A^N_t}$ follows an exogenous process.

Monetary policy follows a Taylor-type rule of the form,

$$\left( \frac{R_t}{R} \right) = \left( \frac{R_{t-1}}{R} \right)^{\theta_R} \left[ \left( \frac{\pi^{NFE}_t}{\pi_t} \right)^{\alpha_{NFE}} \left( \frac{\pi^{1-\alpha_{NFE}}_t}{\pi_t} \right)^{\alpha_{NFE}} \left( \frac{\pi^{GDP/GDP_{t-1}}_t}{a} \right)^{\alpha_Y} \right]^{1-\theta_R} e^{m}_t$$

where the variables without a time subscript are steady state values, $\pi^{NFE}_t$ is core inflation, $GDP_t$ is gross domestic product and $e^{m}_t$ is a monetary shock.

3.6 Foreign Sector

The rest of the world sells the imported inputs goods at price $P^{m,t}_t$ and buys the exported products $Y^X_t$ at the price set by local producers. It is assumed that the goods bought by the foreigners share the same elasticity of substitution as the exportable good bought locally, $\epsilon_X$. In contrast, the demand
for the composite exportable is,
\[ C_{t}^{X*} = \left( \frac{P_{t}^{X}}{S_{t}P_{t}^{*}} \right)^{-\epsilon^{*}} Y_{t}^{*} \xi_{t}^{X*}. \]

Where \( P_{t}^{*} \) is the external CPI index, \( Y_{t}^{*} \) is external demand,\(^{20}\) and \( \xi_{t}^{X*} \) is a disturbance to external demand; all of them assumed to be exogenous stochastic processes.

The closing device of the model is given by the equation for the international interest rate,
\[ R_{t}^{*} = R_{t}^{W} \exp \left\{ \phi_{B} \left( b - \frac{S_{t}B_{t}^{1}}{P_{t}^{1} GDP_{t}} \right) \right\} \xi_{t}^{R1} \xi_{t}^{R2}. \] \(^{(9)}\)

In this way, the external rate relevant for the country is composed of three parts. \( R_{t}^{W} \) represents the world interest rate (which in the data is matched with the LIBOR rate). The term \( \exp \left\{ \phi_{B} \left( b - \frac{S_{t}B_{t}^{1}}{P_{t}^{1} GDP_{t}} \right) \right\} \xi_{t}^{R1} \) represents the country premium (equal to the EMBI Chile in the sample), where \( \xi_{t}^{R1} \) is an exogenous shock.\(^{21}\) Finally, \( \xi_{t}^{R2} \) is a risk-premium shock that captures deviations from the EMBI-adjusted uncovered interest parity (UIP).

### 3.7 Driving Forces

The model features a total of 23 exogenous state variables. Those of domestic origin are consumption preferences (\( \xi_{i}^{d} \)), labor supply (\( \xi_{i}^{H,N} \) and \( \xi_{i}^{H,X} \)), stationary productivity (\( z_{t}^{H} \) and \( z_{t}^{X} \)), the growth rate of the long-run trend (\( a_{t} \)), desired markups (\( \xi_{t}^{X} \) and \( \xi_{t}^{M} \)), endowment of commodities (\( y_{t}^{co} \)), the relative prices of Food and Energy (\( p_{t}^{F} \) and \( p_{t}^{E} \)), efficiency of investment (\( u_{t} \)), government consumption (\( g_{t} \)), and monetary policy (\( \epsilon_{t}^{m} \)). In turn foreign driving forces are the world interest rate (\( R_{t}^{W} \)), foreign risk premium (\( \xi_{t}^{R1} \) and \( \xi_{t}^{R2} \)), international prices of commodities (\( P_{t}^{Co} \)), imported goods (\( P_{t}^{M*} \)) and CPI for trade partners (\( P_{t}^{*} \)), demand for exports of \( X \) (\( \xi_{t}^{X*} \)), and GDP trade partners (\( y_{t}^{*} \)). All these process are assumed to be Gaussian in logs. Markup and monetary-policy shocks are i.i.d. All the other, with the exception of international prices, are AR(1) processes independent of each other.

As the model features a balanced growth path, and preferences are such that relative prices are stationary, foreign prices should to co-integrate, growing all of the same long-run rate.\(^{22}\) Define the inflation of foreign CPI as \( \pi_{t}^{j} = \frac{F_{t}^{j}}{F_{t-1}^{j}} \), with steady state value of \( \pi^{*} \). We propose the following model for these prices,
\[ P_{t}^{j} = (\pi^{*}P_{t-1}^{j})^{\Gamma_{j}}(F_{t}^{*})^{1-\Gamma_{j}}u_{t}^{j}, \quad \text{with } \Gamma_{j} \in [0,1), \quad \text{for } j = \{Co*, M*, *\}, \] \(^{(10)}\)
\[ \Delta F_{t}^{*} = \frac{F_{t}^{*}}{F_{t-1}^{*} \pi^{*}} \exp\left( \frac{\Delta F_{t-1}^{*}}{\pi^{*}} \right)^{\rho F_{t}^{*}}, \quad \text{with } \rho F_{t}^{*} \in (-1,1) \] \(^{(11)}\)
\[ u_{t}^{j} = \left( u_{t-1}^{j} \right)^{\rho_{j}} \exp\left( \epsilon_{t}^{j} \right), \quad \text{with } \rho_{j} \in (-1,1), \quad \text{for } j = \{Co*, M*, *\}, \] \(^{(12)}\)

where \( \epsilon_{t}^{i} \) are i.i.d. \( N(0, \sigma_{t}^{2}) \) for \( i = \{Co*, M*, *, F*\} \).

Under this specification, each price is driven by two factors: a common trend affecting all prices (\( F_{t}^{*} \)) and a price-specific shock (\( u_{t}^{j} \)). The parameter \( \Gamma_{j} \) determines how slowly changes in this trend affect each price. The presence of a common trend generates co-integration among prices (as

\(^{20}\)We assume \( y_{t}^{i} = \frac{Y_{t}^{i}}{Y_{t-1}^{i}} \) follows an exogenous process

\(^{21}\)Here \( GDP_{t} \) denotes growth domestic product and \( P_{t}^{*} \) is the GDP deflator.

\(^{22}\)In other words, the co-integration vector between the log of any pair of these prices should be \((1, -1)\).
long as $\Gamma_j < 1$), and the fact that the exponent in the trend and in the lagged price in (10) add-up to one forces relative prices to remain constant in the long run.\(^{23}\) The usual assumption for these prices in DGSE models with nominal rigidities is obtained as a restricted version of this setup, imposing $\Gamma_j = 0$ for $j = \{Co^*, M^*\}$ and $\sigma^2 = 0$. In other words, the relative prices of both commodities and imports are driven by stationary AR(1) processes, while the inflation of commercial partners is a stationary AR(1) process. The specification in (10)-(12) generalizes this usual assumption in several dimensions. First, in the usual set up, the common trend of all prices is exactly equal to the CPI of commercial partners. This might lead to the wrong interpretation that inflation of commercial partners is an important driver of domestic variables, while in reality this happens because it represents a common trend in all prices. Second, the usual specification imposes that every change in the common trend has a contemporaneous one-to-one impact in all prices, while in reality different prices may adjust to changes in this common trend at different speeds. Finally, for our specific sample the data favors the general specification (10)-(12) relative to the restricted model.

Overall, the model features 24 exogenous disturbances, related to the 23 exogenous state variables previously listed plus the common trend in international prices.

### 3.8 Parametrization Strategy

The values for parameters in the model are assigned by a combination of Calibration and Estimation. The resulting values are presented in the tables of Appendix C. Parameters representing shares in the different aggregate baskets and production functions are calibrated by looking at input output matrices for Chile. We also target several steady-state ratios by using sample averages of the observables counterparts, and draw from related studies estimating DSGEs model for Chile for some parameters that are not properly identified with our data set. Finally, the parameters characterizing the dynamics of some external driving forces are calibrated by estimating AR(1) processes by OLS with the observable counterpart.

The rest of the parameters are estimated using a Bayesian approach. The sample is quarterly from 2001.Q3 to 2016.Q3, and the following 23 series are used:\(^{24}\)

- Real growth rate of: $GDP$, $GDP^X$ (Defined as Agriculture, Fishing, Industry, Utilities, Transportation), $GDP^N$ (Construction, Retail, Services), $GDP^{Co}$ (Mining), private consumption ($C$), total investment ($C$), and government consumption ($G$).
- The ratio of the nominal trade balance to GDP.
- Quarterly CPI-based inflation of $\pi^N$ (services, excluding Food and Energy), $\pi^T$ (goods. ex. Food and Energy), $\pi^M$ (imported goods, ex. Food and Energy), $\pi^F$ (Food) and $\pi^E$ (Energy).
- The growth of nominal wages ($\pi^{WX}$ and $\pi^{WN}$) measured as the cost per unit of labor (the CMO index), using sectors consistent with the GDP’s definition.
- The nominal dollar exchange-rate depreciation ($\pi^S$) and the monetary policy rate ($R$).

\(^{23}\)If $\Gamma_j = 1$, each price is a random walk with a common drift $\pi^*$. Although this implies that in the long run all prices will grow at the same rate, they will not be co-integrated and relative prices may be non-stationary.

\(^{24}\)The source is the Central Bank of Chile. Variables are seasonally adjusted using the X-11 filter, expressed in logs, multiplied by 100, and demeaned. All growth rates are changes from two consecutive quarters.
• External: World interest rate \( (R^W, \text{LIBOR}) \), country premium (EMBI Chile), foreign inflation \( (\pi^*, \text{inflation index for commercial partners, the IPE Index}) \), inflation of Commodities prices \( (\pi^{Cos}, \text{Copper price}) \) and imports \( (\pi^{M*}, \text{using a price index for imported of goods, the IVUM index}) \), external GDP \( (Y^*, \text{measured as GDP for commercial partners}) \).

All domestic observables are assumed to have a measurement error, with calibrated variance equal to 5% of the observable. When possible, priors are set centering the distributions around previous results in the literature. Priors and posteriors are shown in Appendix C. The estimated model is able to properly match the volatilities and first-order autocorrelation coefficients of the domestic observables, as can be seen in Table 1.

### Table 1: Second Moments in the Data and in the Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>St. Dev. (%)</th>
<th>AC(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta GDP )</td>
<td>0.9 (0.1)</td>
<td>1.1</td>
</tr>
<tr>
<td>( \Delta CONS )</td>
<td>1.0 (0.1)</td>
<td>0.8</td>
</tr>
<tr>
<td>( \Delta INV )</td>
<td>3.9 (0.4)</td>
<td>4.4</td>
</tr>
<tr>
<td>( \Delta GDP^X )</td>
<td>1.5 (0.1)</td>
<td>1.5</td>
</tr>
<tr>
<td>( \Delta GDP^N )</td>
<td>0.8 (0.1)</td>
<td>1.6</td>
</tr>
<tr>
<td>( TB/GDP )</td>
<td>5.5 (0.5)</td>
<td>5.2</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.7 (0.1)</td>
<td>0.6</td>
</tr>
<tr>
<td>( \pi^T )</td>
<td>0.7 (0.1)</td>
<td>0.8</td>
</tr>
<tr>
<td>( \pi^N )</td>
<td>0.4 (0.1)</td>
<td>0.4</td>
</tr>
<tr>
<td>( \pi^M )</td>
<td>0.8 (0.1)</td>
<td>0.8</td>
</tr>
<tr>
<td>( \pi^{WX} )</td>
<td>0.6 (0.1)</td>
<td>0.7</td>
</tr>
<tr>
<td>( \pi^{WN} )</td>
<td>0.4 (0.0)</td>
<td>0.4</td>
</tr>
<tr>
<td>( R )</td>
<td>0.4 (0.0)</td>
<td>0.6</td>
</tr>
<tr>
<td>( \pi^S )</td>
<td>5.2 (0.8)</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Note: The variables are: the growth rates of GDP, private consumption, investment, and GDP in the X and N sectors, the trade-balance-to-output ratio, inflation for total CPI, tradables, non-tradables and imported, the growth rate of nominal wages in sector X and N, the monetary policy rate, and the nominal depreciation. Columns two to four correspond to standard deviations, while five to seven are first-order autocorrelations. For each of these moments, the three columns shown are: the point estimate in the data, GMM standard-errors in the data, and unconditional moment in the model at the posterior mode.

#### 3.9 Main Drivers of the NER and Implied Dynamics

As a prelude for the ERPT computations, we discuss the main drivers of the NER according to the estimated model, and explore the intuition behind the effects of these shocks. Table 2 show the contribution of several shocks to explain the unconditional variance of the NER depreciation \( (\pi^S) \); focusing on those that are the main drivers of this variable. In addition, we also show this variance decomposition for alternative inflation measures, the policy rate and the real exchange rate.

As can be seen, the most important shock to account for NER fluctuations is that associated with the trend in international prices \( (\Delta F^*) \), explaining almost 70% of the variance of \( \pi^S \). The risk shock that emerges as deviations from the interest parity (labeled as \( UIP, \xi^{R2} \) in (9)), as well as the world interest rate, explain also a non trivial part of the volatility of \( \pi^S \). Together the three account for
Table 2: Variance Decomposition

<table>
<thead>
<tr>
<th>Var.</th>
<th>MP</th>
<th>RW</th>
<th>CP</th>
<th>UIP</th>
<th>ΔF*</th>
<th>Sum.</th>
</tr>
</thead>
<tbody>
<tr>
<td>πN</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>13</td>
<td>67</td>
<td>94</td>
</tr>
<tr>
<td>π</td>
<td>3</td>
<td>12</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>31</td>
</tr>
<tr>
<td>πF</td>
<td>4</td>
<td>19</td>
<td>5</td>
<td>9</td>
<td>14</td>
<td>50</td>
</tr>
<tr>
<td>πM</td>
<td>3</td>
<td>17</td>
<td>5</td>
<td>8</td>
<td>13</td>
<td>46</td>
</tr>
<tr>
<td>πN</td>
<td>2</td>
<td>13</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td>R</td>
<td>18</td>
<td>18</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>56</td>
</tr>
<tr>
<td>rer</td>
<td>3</td>
<td>15</td>
<td>4</td>
<td>11</td>
<td>15</td>
<td>48</td>
</tr>
</tbody>
</table>

Note: Each entry shows the % percentage of the asymptotic variance of the variable in each row, explained by the shock in each column, computed at the posterior mode. The shocks correspond to monetary policy (MP), world interest rate (RW), country premium (CP), deviations from UIP (UIP) and the trend in international prices (ΔF*).

almost 90% of the variance of the NER. These shocks play a non trivial role in accounting for inflation variability, explaining around 40% of tradable inflation, 20% of non-tradables, and 25% of total CPI. Thus, while clearly not the only relevant factors, the determinants of the NER are important to determine inflation fluctuations as well. This similarity is also reflected in the decomposition of the policy rate (that responds to inflation movements according to the rule) and the real exchange rate.

Given the importance of these shocks, we now discuss the dynamics they generate in the economy. The trend to international prices propagates domestically through two main channels. Suppose there is a negative shock to this trend. First, by lowering the price of commodities and inflation in commercial partners (which is the reference price to determine the demand for exports of X goods), it generates a drop on export-related income. At the same time, by reducing the price of imports abroad, it puts downward pressure to imported inflation domestically. While the terms-of-trade measured at international prices might not move significantly, the effect on exports dominates at home as domestic import prices will change only gradually due to price rigidities.

The second channel is a valuation effect in international debt. As foreign bonds are denominated in dollars, an unexpected drop in foreign prices will increase, ceteris paribus, the burden of interest payments from debt carried from the previous period, in domestic currency units. At the same time, as the shock is persistent, everything else equal, the real rate associated with foreign borrowing will increase. Both of them generate a contraction domestically if the economy has a negative foreign asset position; as we assume in our calibration.

These two channels reinforce each other, leading to the responses in Figure 1. After a negative shock to the international trend in prices, aggregate demand falls. As the market for non-traded goods has to clear domestically, the shock generates a fall in the relative price of non-tradables, a real exchange rate depreciation, a drop in production in the N sector, an increase in output in the X sector, and an overall fall in GDP. Moreover, given the real depreciation and the presence of price rigidities, the nominal exchange rate depreciates as well.

To explain the dynamics of inflation first notice that the required fall in the relative price of non-tradables would lead to an increases in the price of tradables and a drop in that of non-tradables, which can actually be observed in the very short run in the figure. The rise in the price of tradables

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For instance, if $\Gamma_j = 0$ in (10), the ratio of commodity to import prices will not change.
Figure 1: IRF to a drop in the trend of international prices

Note: Each graph presents the impulse response function, computed at the posterior mode, expressed as percentage deviations relative to the steady-state. The variables are GDP, Consumption, Investment, GDP in the $X$ and the $N$ sectors, total inflation, tradables and non-tradables inflation (excluding Food and Energy), the monetary policy rate, and the nominal exchange rate, the real exchange rate, and the variable being shocked. The size of the shock is equal to one standard-deviation.

also comes from the pressure exhorted by the increase in international prices; leading to a larger positive change in $\pi^T$ relative to the drop in $\pi^N$. However, under the assumption of indexation to aggregate inflation (both for wages and prices), inflation for non-tradables will start to rise after a few periods. Therefore, the indexation channel will be important to explain a ERPT to non-tradable prices, an effect that will appear only gradually. Finally, given the monetary policy rule, the domestic interest rate increases to smooth the increase in inflation.

In Figure 2 we show the responses to the UIP shock. This positive shock works in the same way as a rise in the world interest rate, by inducing an increase in the cost of foreign borrowing. This triggers both income and substitution effects, leading to a contraction in aggregate demand. Thus

---

26 Import prices rise following the increase in international prices, while the domestic price of $X$ goods is increased as the producer will require a larger domestic price to be indifferent between exporting and selling domestically.

27 The importance of aggregate indexation in the pricing rules for wages and prices in the non-traded sector, while positive, is not estimated to be too large. For wages, only 11% of those that cannot freely choose will adjust to aggregate past inflation, and for prices this this fraction is close to 20%. Still, one can numerically show that if these are set to zero, the response of $\pi^N$ will be negative for the relevant horizon.
qualitatively the responses are analogous to those originated by the shock to $\Delta F^*$.

Quantitatively, however, we can see that the typical shock to the UIP generates somehow milder responses in most variable (particularly in the NER) that the typical shock to $\Delta F^*$.

Figure 2: IRF to a positive risk shock (deviations from UIP)

A final point in comparing both shocks is their long-run effect on the price level and the NER. In this model, the real exchange rate (defined as the NER, times the international price level, divided by the domestic price level) is stationary; a results that follows from the neutrality of money and the fact that the marginal rates of substitutions are assumed to be stationary. Thus, except for shocks affecting foreign inflation $\pi^*$, all others require in the long-run an equal change in the price level and in the NER, generating a long-run conditional ERPT of one. In contrast, shocks that move the international price level in the long run do not require an equal response of the domestic price level and the NER. For the case of the $\Delta F^*$ shock, the long-run conditional ERPT is close to 0.2. Moreover, as all relative prices are stationary in the model, this difference will hold for all prices, not only for the aggregate CPI. As the unconditional ERPT will be an average of these two, we can already see how, depending on the shock, the conditional the ERPT might be quite different.

In particular, this implies, as we argued in the introduction, that a VAR identification strategy based on sign-restrictions cannot separate these two, although as we will see they imply very different conditional ERPTs.
4 Conditional vs. Unconditional ERPT

We begin by computing the conditional ERPT associated with the three main shocks behind the fluctuation in the NER. We present the results for aggregate CPI ($P$), tradables ($T$), imported ($M$), and non-tradables ($N$), the last three excluding Food and Energy. In line with the discussion about the long-run behavior, the unconditional ERPTs generated by $\Delta F^*$ is significantly different from those implied by the shocks to the UIP and to the world interest rate $R^W$. For a horizon of 2 years, the conditional ERPT given a shock to international prices is less than 0.1 for total CPI, smaller than 0.05 for non-tradables, and close to 0.15 for both tradables and importables.

In sharp contrast, for the same horizon, the conditional ERPT to the UIP shocks is much larger for all prices: close to 0.5 for CPI, larger than 0.8 for tradables and imported, and near 0.2 for non-tradables. For the world-interest-rate shock the conditional ERPT are somehow smaller, but still quite larger than those obtained after a shock in the trend of international prices. Clearly, a decision maker at a Central Bank should worry about the source of the shock generating the exchange rate movement in assessing the likely effect on prices.

The empirical VAR literature using Chilean data estimates an ERPT close to 0.2% for total CPI after two years, with a similar value for tradables and close to 0.05 for non-tradables,\(^{29}\) This value is clearly between the conditional ERPTs we just discussed. However, we want to see if the model can generate an unconditional ERPT similar to those estimates in the literature, to see if our model can reach the same conclusions when discussing similar objects. To that end, Figure 4 displays both measures of unconditional ERPT we introduced in Section 2: panel A shows the weighted average of conditional ERPTs as in (8), while panel B displays that obtained using the Population VAR approach.\(^{30}\)

\(^{29}\)See, for instance, Justel and Sansone (2015), Contreras and Pinto (2016), Albagli et al. (2015), among others.

\(^{30}\)The VAR is assumed to contain the following variables: World interest rate ($R^W$), foreign inflation ($\pi^*$), inflation.
A first point to notice is that both measures generate similar unconditional ERPTs, with the exception of non-tradables for which the one from the Population VAR is somehow smaller. Moreover, these values are quite close to those obtained in the empirical literature. Besides this comparison, we can see how these unconditional measures are a weighted average of the conditional discussed before, some how closer to the conditional ERPT for $\Delta F^*$ than for the others. In particular, we can numerically compute the long-run ERPT, which for the measure $UERP_T^M(h)$ is close to 0.3, while for the $UERP_T^{PV}(h)$ is around 0.2.

Overall, the evidence presented in this section points to the fact that the conditional ERPTs are quite different from those obtained from aggregate ERPT measures comparable to those in the literature. Thus, using the results from the empirical literature will lead to a bias in the likely effect of inflation after a large movement in the NER, an estimate that could be greatly improved by an assessment of which shock is behind the particular NER change.

5 ERPT and Expected Monetary Policy

Our second concern regarding the use of the ERPT obtained from the empirical literature is that it could mistakenly lead to think that actual and future monetary policy has little to say about the behavior of both the NER and prices. Conceptually, this point is independent from the potential differences between conditional and unconditional ERPT; although we will see that quantitatively the source of the shock also matters for this discussion. In this section, we design an experiment aimed at...
quantifying whether this concern might be relevant.

The starting point is to notice that, as discussed in Section 3.9, in the benchmark model the monetary policy rate increases (and it is expected to remain high) in response to the main shocks that depreciates the currency. This is already a point of departure with the practice of using empirical ERPT measures for policy analysis. For the response of the policy rate implicit in the ERPT coefficient obtained with a VAR or a single-equation approach cannot be made explicit. What a staff member should tell to the Board at a Central Bank is something along these lines: “this is the expected ERPT if you move the interest rate in this particular way; if not, the ERPT might differ.” Unfortunately, the empirical literature cannot provide such an assessment conditional on the expected path of monetary policy; but it can be done with a DSGE model.

We compare the benchmark ERPT, obtained with a path for the policy rate that follows the estimated rule, with alternatives scenarios that temporarily deviates from the estimated rule. In particular, at the moment that the shock hits the economy, the central bank announces that it will maintain the interest rate at its pre-shock level for $N$ periods, and afterwards the rate will be determined by the estimated rule.\footnote{Computationally, this is implemented by a backward-looking solution as in \textcite{KulishPagan2016} or the appendix in \textcite{Garcia-Cicco2011}.}

Figure 5: IRF under alternative policy paths

A. Trend to international prices

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart1.png}
\caption{IRF under alternative policy paths}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart2.png}
\caption{IRF under alternative policy paths}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart3.png}
\caption{IRF under alternative policy paths}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart4.png}
\caption{IRF under alternative policy paths}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart5.png}
\caption{IRF under alternative policy paths}
\end{figure}

Note: The solid-blue line represents the benchmark case (when the policy rate follows the estimated rule) the dashed-red line is the case in which the rate is fixed for two periods, and the dashed-dotted-black line is when the rate is fixed for 4 periods. The variables shown are the policy rate, total, total, tradable and non-tradable inflations, and the nominal exchange rate.

Figure 5 shows how the impulse-response function changes with these policy alternatives, for the main shocks that drive the NER. Relative to the baseline, these alternatives are more dovish, implying more inflation in all goods. At the same time, a relatively lower policy rate path implies, by the interest
rate parity, a more depreciated NER. Thus, as the ERPT is the ratio of the response of a price and the exchange rate, it is not obvious how it will change with these alternative policy paths.

Using these responses, Figure 6 shows the conditional ERPTs for these policy alternatives. When the shock to the trend in international prices hits the economy, the conditional ERPT varies significantly depending on the reaction of monetary policy. For instance, after two years, the ERPT for total CPI can almost double if the policy rate remains fixed for a year; and the difference can be even larger for non-tradables. At the same time, conditional on shocks to either the UIP or the world interest rate, the ERPT measures do not seem to vary significantly as monetary policy changes; except for non-tradables where we can see some differences.

Figure 6: Conditional ERPT, under alternative policy paths

Note: See Figure 5

In Figure 7 we compute the unconditional ERPT using the weighted average of conditional ones as
As can be seen, influenced mainly by the behavior of the ERPT after the shock to international prices, the unconditional ERPT also increases with a more dovish policy. This comparison provides yet another reason to properly account for the source of the shock and to compute conditional ERPTs, as the effect of alternative policy paths will be relevant depending on the shock.

In sum, this analysis highlights that, in thinking about how monetary policy should react to shocks that deprecates the currency, a menu of policy options and their associated conditional ERPT should be analyzed. Depending on the shock, monetary policy might have an important role to determine the final outcome of both inflation and the NER, although this might not be the case for some relevant shocks that might hit the economy. However, this kind of analysis cannot be performed using the tools and results from the empirical literature literature.

6 Conclusions

This paper was motivated by the widespread use for monetary-policy analysis of results regarding the ERPT generated by empirical, reduced-form methodologies. We analyzed two potential problems: that the ERPT might be different depending on the shock hitting the economy (separating conditional and unconditional ERPT), and that the ERPT might depend on the expected path of monetary policy. We first established the relationship between the ERPT measures used in the empirical literature with related objects obtained from a general equilibrium model. We then used a DSGE model estimated with Chilean data to quantitatively justify that these distinctions are indeed relevant, and that a policy maker using the results from the empirical literature without being aware of these shortcomings might be deciding using inappropriate tools.

Another way to frame this discussion is the following. From the point of view of general equilibrium models, one can define alternative measures of what “optimal” policy means and then fully characterize how monetary policy should respond to particular shocks hitting the economy, in order to achieve the optimality criteria. In that discussion, structural parameters, the role of expectation formation, the nature of alternative driving forces, among other important details, will be relevant to determine the path that monetary policy should follow. However, as the empirical measure of the ERPT computed in the literature is, in one way or another, a conditional correlation and not a structural characteristic of the economy, all the relevant aspects of optimal monetary policy can be described without using the concept of ERPT at all. Thus, while the results of the empirical literature can be useful for other important discussions in International Macroeconomics, its relevance for monetary policy analysis is much more limited.

\[^{33}\]In this computation, we exclude the monetary policy shock in all models, as it plays no role once we fix the policy rate, and we maintain the weights as in the baseline to isolate the changes only due to different dynamics with alternative policy paths. Moreover, the Population VAR measure of aggregate ERPT will not vary with this policy comparison, as the alternative paths for the interest rate will only affect the dynamics in the short run, without changing the population moments.
7 References


A ERPT in VARs and DSGE Models

A.1 The 2x2 Case

Consider the VAR representation of a DSGE model for the (log of the) nominal depreciation and the inflation of good $j$ of interest (i.e. $x_t = [\pi^S_t, \pi_j^t]'$) that takes the simple form

$$x_t = F x_{t-1} + Q e_t,$$  \hfill (13)

where $e_t$ is a vector of two i.i.d. structural shocks, with mean zero and variance equal to the identity matrix. With this model, the response of a variable $k$ to a shock $i$, $h$ periods after the shock, is given by the element $k, i$ of the matrix $F^h Q$.

As the variables in $x_t$ are first differences of the two variables of interest, the impulse response of the variables in levels is obtained by the cumulative sum of the impulse response of of the vector $x_t$. Thus, the ERPT in the DSGE model, conditional on the shock $i$, $h$ periods ahead, is given by

$$CERP^M_{i}(h) = \frac{CIRF^M_{k,i}(h)}{CIRF^M_{1,i}(h)} = \left[ \left( \sum_{t=0}^{h} F^t \right) Q \right]_{1i}^{2i},$$

where, for a generic matrix $A$, $A_{ki}$ denotes the element in the $k$th row and $i$th column. Let $F(h) = \left( \sum_{t=0}^{h} F^t \right)$. Thus, we have

$$CERP^M_{i}(h) = \frac{F(h)_{21} Q_{1i} + F(h)_{22} Q_{2i}}{F(h)_{11} Q_{1i} + F(h)_{12} Q_{2i}}.$$  \hfill (14)

Additionally, the forecast error variance $h$ periods ahead, of the level of variable $k$ due to movements in the shock $i$ is given by

$$FEV^M_{k,i}(h) = \sum_{t=0}^{h} (CIRF^M_{k,i}(h))^2 = \sum_{t=0}^{h} (F(t)_{k1} Q_{1i} + F(t)_{k2} Q_{2i})^2$$

and, expressed in percentage terms of the total forecast error variance we have,

$$FEV^D_{k,i}(h) = \frac{FEV^M_{k,i}(h)}{FEV^M_{1,i}(h) + FEV^M_{k,2}(h)}.$$

Also, notice that when $h = 0$,

$$FEV^M_{j,i}(0) = (Q_{ji})^2, \quad FEV^D_{j,i}(0) = \frac{(Q_{ji})^2}{(Q_{1j})^2 + (Q_{2j})^2}.$$  \hfill (15)

At the same time, as discussed in subsection 2.1, someone from the VAR literature using a large sample generated by this simple DSGE model will estimate a VAR(1),

$$y_t = \Phi y_{t-1} + u_t,$$  \hfill (15)

obtaining the estimates $\Phi = F$ and $\Omega = QQ'$. Then, she will proceed by applying a Cholesky...
decomposition to \( \Omega \) and estimating the ERPT as in (4). In this simple case, this implies

\[
ERPT^V(h) = \frac{CIRF^V_{2,1}(h)}{CIRF^V_{1,1}(h)} = \frac{\left( \sum h \Phi^h \right) P_{2,1}}{\left( \sum h \Phi^h \right) P_{1,1}},
\]

or, using the fact that \( \Phi = F \),

\[
ERPT^V(h) = \frac{F(h)_{21}P_{11} + F(h)_{22}P_{21}}{F(h)_{11}P_{11} + F(h)_{12}P_{21}}. \tag{16}
\]

To obtain the link between the VAR-based ERPT and the conditional ERPTs in the DSGE we need to characterize how \( P \) and \( Q \) are related. Given the simple 2 \( \times \) 2 structure of these matrices,

\[
\begin{bmatrix}
\Omega_{11} & \Omega_{12} \\
\Omega_{21} & \Omega_{22}
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{21} & Q_{22}
\end{bmatrix} \begin{bmatrix}
Q_{11} & Q_{21} \\
Q_{12} & Q_{22}
\end{bmatrix} = \begin{bmatrix}
Q_{11}^2 + Q_{12}^2 & Q_{11}Q_{21} + Q_{12}Q_{22} \\
Q_{21}Q_{11} + Q_{22}Q_{12} & Q_{21}^2 + Q_{22}^2
\end{bmatrix}
\]

Additionally, the Cholesky factorization of matrix \( \Omega \) is such that,\(^{34}\)

\[
\begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} = \begin{bmatrix}
Q_{11}^{1/2} & 0 \\
\Omega_{21}Q_{11}^{-1/2} & \left( \Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12} \right)^{1/2}
\end{bmatrix} = \ldots
\]

\[
\begin{bmatrix}
(Q_{11}^2 + Q_{12}^2)^{1/2} & 0 \\
(Q_{21}Q_{11} + Q_{22}Q_{12})(Q_{11}^2 + Q_{12}^2)^{-1/2} & (Q_{21}^2 + Q_{22}^2 - (Q_{21}Q_{11} + Q_{22}Q_{12})^2(Q_{11}^2 + Q_{12}^2)^{-1})^{1/2}
\end{bmatrix}
\]

With these formulas, we can write (16) as

\[
ERPT^V(h) = \frac{F(h)_{21}(Q_{11}^2 + Q_{12}^2)^{1/2} + F(h)_{22}(Q_{21}Q_{11} + Q_{22}Q_{12})(Q_{11}^2 + Q_{12}^2)^{-1/2}}{F(h)_{11}(Q_{11}^2 + Q_{12}^2)^{1/2} + F(h)_{12}(Q_{21}Q_{11} + Q_{22}Q_{12})(Q_{11}^2 + Q_{12}^2)^{-1/2}}
\]

\[
= \frac{F(h)_{21}(Q_{11}^2 + Q_{12}^2) + F(h)_{22}(Q_{21}Q_{11} + Q_{22}Q_{12})}{F(h)_{11}(Q_{11}^2 + Q_{12}^2) + F(h)_{12}(Q_{21}Q_{11} + Q_{22}Q_{12})}
\]

\[
= \frac{(F(h)_{21}Q_{11} + F(h)_{22}Q_{21})Q_{11} + (F(h)_{21}Q_{12} + F(h)_{22}Q_{22})Q_{12}}{(F(h)_{11}Q_{11} + F(h)_{12}Q_{21})Q_{11} + (F(h)_{11}Q_{12} + F(h)_{12}Q_{22})Q_{12}}
\]

\[
= \frac{CERPT^V_{1}(h) [F(h)_{11}Q_{11} + F(h)_{21}Q_{21}]Q_{11} + CERPT^V_{2}(h) [F(h)_{12}Q_{22} + F(h)_{22}Q_{22}]Q_{12}}{[F(h)_{11}Q_{11} + F(h)_{21}Q_{21}]Q_{11} + [F(h)_{12}Q_{22} + F(h)_{22}Q_{22}]Q_{12}}
\]

\[
= \frac{CERPT^V_{1}(h)CIRF^M_{1,1}(h)Q_{11} + CERPT^V_{2}(h)CIRF^M_{1,2}(h)Q_{12}}{CIRF^M_{1,1}(h)Q_{11} + CIRF^M_{1,2}(h)Q_{12}}. \tag{17}
\]

\[
= \frac{CERPT^V_{1}(h)CIRF^M_{1,1}(h)Q_{11}(Q_{11})^2 + CERPT^V_{2}(h)CIRF^M_{1,2}(h)Q_{12}(Q_{12})^2}{CIRF^M_{1,1}(h)Q_{11}(Q_{11})^2 + CIRF^M_{1,2}(h)Q_{12}(Q_{12})^2}. \tag{18}
\]

If we define,

\[
\omega_i(h) = \frac{CIRF^M_{i,1}(h)Q_{11}(Q_{11})^2}{CIRF^M_{1,1}(h)Q_{11}(Q_{11})^2 + CIRF^M_{1,2}(h)Q_{12}(Q_{12})^2},
\]

\(^{34}\)See Hamilton (1994), Section 4.4.
we obtain equation (7) in the text. For \( h = 0 \), the expression simplifies to
\[
\omega_i(0) = \frac{(Q_{11})^2}{(Q_{11})^2 + (Q_{12})^2} = FEV D^M_{i,1}(0),
\]
In other words, the weight at \( h = 0 \) corresponds to the fraction of the forecast-error variance of the NER explained by shock \( i \). For \( h > 0 \), the forecast-error variance is adjusted by the ratio of the response of the NER at period \( h \) relative to that at \( h = 0 \).

Based on this analysis, we propose the following generalization for a any DSGE model. Let \( k \) be the position of the inflation of interest in the vector \( y_t \), and let \( s \) be the position of the nominal depreciation rate in the vector \( y_t \). The DSGE-based unconditional ERPT is defined as,
\[
UERP T^M_k(h) \equiv \sum_{i=1}^{n_e} CERP T^M_{k,i}(h) \omega_i(h),
\]
where \( n_e \) is the number of shock in the vector \( e_t \), \( CERP T^M_{k,i}(h) \) is computed as in (6), and
\[
\omega_i(h) = \frac{\text{CIRF}^M_{s,i}(h)Q_{si}^2}{\sum_{l=1}^{n_e} \text{CIRF}^M_{s,l}(h)Q_{sl}^2}.
\]

A.2 ERPT From the Population VAR

From the linearized solution of the DGSE model (5), provided stationarity, the variance-covariance matrix \( \Sigma_0 = E(y_t y_t') \) satisfies,
\[
\Sigma_0 = F \Sigma_0 F' + QQ', \tag{20}
\]
which can be easily computed. In addition, the matrix containing the auto-covariance of order \( p \) is \( \Sigma_p = E(y_t y_{t-p}) = F^p \Sigma_0 \) for \( p > 0 \). Finally, we are interested in subset \( x_t \) of \( n \) variables from \( y_t \), that will be included in the VAR model, defined as \( x_t \equiv Sy_t \) for an appropriate choice of \( S \). In that case, we have
\[
E(x_t x_{t-p}') = SE(y_t y_{t-p})S' = S \Sigma_p S'. \tag{21}
\]
for \( p \geq 0 \).

The structural VAR(p) model for the vector \( x_t \) in (2)-(3) can be written in more compact form, defining the vector \( X_t = [x'_t \: x'_{t-1} \: x'_{t-p+1}]' \), in two alternative ways. Either,
\[
x_t = \Phi X_{t-1} + Pw_t, \tag{22}
\]
where \( \Phi = [\Phi_1 \: \ldots \: \Phi_p] \) or,
\[
X_t = \tilde{\Phi} X_{t-1} + U_t, \tag{23}
\]
where,
\[
\tilde{\Phi} = \begin{bmatrix} \Phi \\ I_{n(p-1)} \end{bmatrix}, \quad U_t = \tilde{P} w_t, \quad \tilde{P} = \begin{bmatrix} P \\ 0_{n(p-1) \times n} \end{bmatrix}.
\]
Using (23) the IRF of the variables in the position \( j \) in the vector \( x_t \) to the shock associated with the
\[^{35}\text{For instance, } \text{vec}(\Sigma_0) = (I - F \otimes F)^{-1} \text{vec}(QQ').\]
variable in the position \( i \), \( h \) periods after the shock, is is given by the \( \{ j, i \} \) element of the matrix \( \tilde{\Phi}^h \tilde{P} \). The cumulative IRF is just the element \( \{ j, i \} \) in the matrix \( \sum_{s=0}^{h} \tilde{\Phi}^s \tilde{P} \).

An econometrician would proceed by choosing a lag order \( p \) in the VAR and estimate (22) by OLS. If she had available an infinite sample, she can estimate (22) using the population OLS; i.e. choosing \( \tilde{\Phi} \) to minimize,

\[
E \left[ (x_t - \tilde{\Phi} X_{t-1})' (x_t - \tilde{\Phi} X_{t-1}) \right].
\]

This is equivalent to \( \tilde{\Phi} \) satisfying the first order condition,

\[
E \left[ (x_t - \tilde{\Phi} X_{t-1}) X_{t-1}' \right] = 0,
\]

which can be solved to obtain,

\[
\tilde{\Phi} = E \left( x_t X_{t-1}' \right) \left[ E \left( X_{t-1} X_{t-1}' \right) \right]^{-1}, \tag{24}
\]

Similarly,

\[
\tilde{\Omega} = E(u_t u_t') = E \left[ (x_t - \tilde{\Phi} X_{t-1})(x_t - \tilde{\Phi} X_{t-1})' \right] \\
= E \left( x_t x_t' \right) + \tilde{\Phi} E \left( X_{t-1} X_{t-1}' \right) \tilde{\Phi}' - E \left( x_t X_{t-1}' \right) \tilde{\Phi}' - \tilde{\Phi} E \left( X_{t-1} x_t' \right) \\
= E \left( x_t x_t' \right) + E \left( x_t X_{t-1}' \right) \left[ E \left( X_{t-1} X_{t-1}' \right) \right]^{-1} E \left( X_{t-1} x_t' \right) - \\
E \left( x_t X_{t-1}' \right) \left[ E \left( X_{t-1} X_{t-1}' \right) \right]^{-1} E \left( X_{t-1} x_t' \right) - E \left( x_t X_{t-1}' \right) \left[ E \left( X_{t-1} X_{t-1}' \right) \right]^{-1} E \left( X_{t-1} x_t' \right) \\
= E \left( x_t x_t' \right) - E \left( x_t X_{t-1}' \right) \left[ E \left( X_{t-1} X_{t-1}' \right) \right]^{-1} E \left( X_{t-1} x_t' \right) = E \left( x_t x_t' \right) - \tilde{\Phi} E \left( X_{t-1} x_t' \right) \tag{25}
\]

In most applied cases, with finite samples, econometricians estimate the parameters of the VAR and use asymptotic theory to derive probability limits and limiting distributions to perform inference,\textsuperscript{36} such as hypothesis testing or computing confidence bands. The case we want to analyze here is different, as we assume the DSGE model is the true data generating process, and we wish to compute the model that an econometrician would estimate with an infinite or population sample. This is equivalent to compute \( \tilde{\Phi} \) and \( \tilde{\Omega} \) in (24)-(25) using the population moments from the DSGE.

Given \( x_t = S y_t \), and recalling the definition of \( X_t \), we have,

\[
E \left( x_t x_t' \right) = S \Sigma_0 S',
\]

\[
E \left( x_t X_{t-1}' \right) = \left[ E \left( x_t x_{t-1}' \right) \ E \left( x_t x_{t-2}' \right) \ ... \ E \left( x_t x_{t-p}' \right) \right] = \left[ S \Sigma_1 S' \ S \Sigma_2 S' \ ... \ S \Sigma_p S' \right]
\]

\textsuperscript{36}For instance, (24) and (25) are the probability limits of the OLS estimators for \( \Phi \) and \( \Omega \), by virtue of both the Law of Large Numbers and the Continuous Mapping Theorem.
\[
E \left( X_{t-1} X'_{t-1} \right) = \begin{bmatrix}
E \left( x_{t-1} x'_{t-1} \right) & E \left( x_{t-1} x'_{t-2} \right) & \cdots & E \left( x_{t-1} x'_{t-p} \right) \\
E \left( x_{t-2} x'_{t-1} \right) & E \left( x_{t-2} x'_{t-2} \right) & \cdots & E \left( x_{t-2} x'_{t-p} \right) \\
\vdots & \vdots & \ddots & \vdots \\
E \left( x_{t-p} x'_{t-1} \right) & E \left( x_{t-p} x'_{t-2} \right) & \cdots & E \left( x_{t-p} x'_{t-p} \right)
\end{bmatrix} = \begin{bmatrix}
E_{X1} & E_{X2} & \cdots & E_{Xp} \\
E_{X2} & E_{X3} & \cdots & E_{Xp} \\
\vdots & \vdots & \ddots & \vdots \\
E_{Xp} & E_{Xp+1} & \cdots & E_{Xp+p-1} 
\end{bmatrix} = \begin{bmatrix}
E_{X1}^{\prime} & E_{X2}^{\prime} & \cdots & E_{Xp}^{\prime} \\
E_{X2}^{\prime} & E_{X3}^{\prime} & \cdots & E_{Xp}^{\prime} \\
\vdots & \vdots & \ddots & \vdots \\
E_{Xp}^{\prime} & E_{Xp+1}^{\prime} & \cdots & E_{Xp+p-1}^{\prime}
\end{bmatrix}
\]

which are all the elements required to compute \( \hat{\Phi} \) and \( \hat{\Omega} \).

A final comment relating the usual practice in the VAR literature. In most papers the vector \( x_t \) contains foreign variables. If the assumption of a small and open economy is used, it is generally assumed that the matrices \( \Phi_j \) for \( j = 1, \ldots, p \) are block lower triangular: i.e. lags of domestic variables cannot affect foreign variables. In practice, this second constraint is implemented by estimating the matrices \( \Phi_j \) by FGLS o FIML, applying the required restrictions. Here, however, if the DSGE model assumes that foreign variables cannot be affected by domestic variables, the auto-covariance matrices \( \Sigma_j \) will have zeros in the appropriate places, so that \( \hat{\Phi} \) will display the same zero constrains the econometrician would impose.

B Quantitative DSGE Model Appendix

B.1 Optimality Conditions

B.1.1 Household

From the decision of final consumption, labor, bonds and capital and defining as \( \lambda_t \) the multiplier of the budget constraint, \( \mu_t^J \lambda_t \) the multiplier of the capital accumulation equation for \( J = \{X, N\} \) and as \( \mu_t^{WJ} W_t^J \lambda_t \) the multiplier of the equalization of labor demand and supply, we have the first order conditions:

\[
\begin{align*}
\xi^\beta_t (C_t - \phi e \tilde{C}_{t-1})^{-\sigma} & - P_t \lambda_t = 0 \\
-\xi^\beta_t \kappa_t^{h,X} (h_t^X)^{\varphi} + \mu_t^{WX} W_t^X \lambda_t & = 0 \\
-\xi^\beta_t \kappa_t^{h,N} (h_t^N)^{\varphi} + \mu_t^{WN} W_t^N \lambda_t & = 0 \\
-\lambda_t + \beta E_t \lambda_{t+1} R_t & = 0 \\
-\lambda_t S_t + \beta E_t \lambda_{t+1} S_{t+1} R_t^* & = 0 \\
-\mu_t^J \lambda_t + \beta E_t \left\{ \lambda_{t+1} I_{t+1}^J R_{t+1}^J + \mu_t^J \lambda_{t+1} (1 - \delta) \right\} & = 0 \\
-\lambda_t P_t^J + \mu_t^J \lambda_t \left\{ \left( 1 - \Gamma \right) \frac{I_t^J}{I_{t-1}^J} \right\} u_t + \left(-\Gamma' \left( \frac{I_t^J}{I_{t-1}^J} \right) \frac{1}{I_{t-1}^J} \right) u_t I_t^J & = 0 \\
\text{beta} E_t \left\{ \mu_t^{J+1} \lambda_{t+1} \left( -\Gamma' \left( \frac{I_{t+1}^J}{I_t^J} \right) \right) \left(-\frac{I_{t+1}^J}{(I_t^J)^2} \right) u_{t+1} I_{t+1}^J \right\} & = 0
\end{align*}
\]
The last two equations for \( J = \{ X, N \} \). From the optimality conditions of choosing wages, we can write the first order conditions as:

\[
\begin{align*}
\epsilon_W \frac{1}{W_t^J \epsilon_w} & \sum_{\tau=0}^\infty \left( \theta_{W,j}^J \beta_\tau \right) \lambda_{t+\tau} \left\{ \frac{h_{t+\tau}^{J,t}}{(W_t^J)^{1-\epsilon_w}} (W_t^{J,s})^{-\epsilon_w} \left[ a^T \prod_{s=1}^T \left( (\pi_{t+s}^J)_{\pi_{t+s+1}^{J,t}} \pi_{t+s}^{J,t} \right) \right]^{1-\epsilon_w} \right\} = \\
E_t & \sum_{\tau=0}^\infty \left( \theta_{W,j}^J \beta_\tau \right) \mu_{t+\tau} \lambda_{t+\tau} \left\{ \frac{h_{t+\tau}^{J,t}}{(W_t^J)^{1-\epsilon_w}} (W_t^{J,s})^{-\epsilon_w} \left[ a^T \prod_{s=1}^T \left( (\pi_{t+s}^J)_{\pi_{t+s+1}^{J,t}} \pi_{t+s}^{J,t} \right) \right]^{1-\epsilon_w} \right\} 
\end{align*}
\]

For \( J = \{ X, N \} \).

In addition, the optimality conditions for the decision between tradable and non-tradable consumption are:

\[
\begin{align*}
C_t^N & = \gamma \left( \frac{P_t^N}{P_t} \right)^{-\theta} C_t \\
C_t^T & = (1 - \gamma) \left( \frac{P_t^T}{P_t} \right)^{-\theta} C_t
\end{align*}
\]

where it was used the fact that \( C_t^{SAE} = C_t \).

And between the exportable and importable:

\[
\begin{align*}
C_t^X & = \gamma_T \left( \frac{P_t^X}{P_t^T} \right)^{-\theta_T} C_t^T \\
C_t^M & = (1 - \gamma_T) \left( \frac{P_t^M}{P_t^T} \right)^{-\theta_T} C_t^T
\end{align*}
\]

B.1.2 Investment Good

The first order conditions between tradable and non-tradable investment can be written as:

\[
\begin{align*}
\tilde{I}_t^N & = \gamma_I \left( \frac{P_t^N}{P_t^I} \right)^{-\theta_I} I_t \\
\tilde{I}_t^T & = (1 - \gamma_I) \left( \frac{P_t^T}{P_t^I} \right)^{-\theta_I} I_t
\end{align*}
\]

And between exportable and importable investment:

\[
\begin{align*}
\tilde{I}_t^X & = \gamma_{T,I} \left( \frac{P_t^X}{P_t^{T,I}} \right)^{-\theta_{T,I}} \tilde{I}_t^T \\
\tilde{I}_t^M & = (1 - \gamma_{T,I}) \left( \frac{P_t^M}{P_t^{T,I}} \right)^{-\theta_{T,I}} \tilde{I}_t^T
\end{align*}
\]
B.1.3 Firms

The first order conditions are the same for each firm $i$ in each sector and so the subscript will be omitted. First, given the marginal costs, the first order condition of the price setting can be written as:

$$
\frac{\xi^J_{t,e_j} - 1}{e_j} (P_t^J)^{-e_j} \sum_{s=1}^{\infty} (\beta \theta)^J \Lambda_{t,s} \frac{1}{(P_t^J)^{-e_j}} Y_t^J \left[ \prod_{s=1}^{\tau} (\pi_{t+s-1}^{J,e_j} \pi_{t+s-1}^{-1})^{e_j} \right]^{1-e_j} = (P_t^J)^{-e_j-1} \sum_{s=1}^{\infty} (\beta \theta)^J \Lambda_{t,s} \frac{1}{(P_t^J)^{-e_j}} Y_t^J \left[ \prod_{s=1}^{\tau} (\pi_{t+s-1}^{J,e_j} \pi_{t+s-1}^{-1})^{e_j} \right]^{1-e_j}
$$

To get the marginal cost of each sector, we distinguish between the importable and the other sectors

- **Sector M** Cost minimization implies that their marginal cost is the same for all firms and is:
  
  $$MC_t^M = P_{m,t}$$

  Note the difference between the price set by the $M$ sector, $P_t^M$, and the price of its input, $P_{m,t}$.

- **Sector X and N**

  1. Optimal production of $V_t^J$: The first order conditions and the marginal cost are:

  $$h_t^J = \frac{V_t^J}{z_t^J (A_t^J)^{1-\alpha_J} \left[ 1 - \alpha_J (P_t^J R_t^J)^{\alpha_J} \right]}$$
  
  $$K_{t-1} = \frac{V_t^J}{z_t^J (A_t^J)^{1-\alpha_J} \left[ 1 - \alpha_J (P_t^J R_t^J)^{\alpha_J} \right]}$$
  
  $$MC_t^{V,J} = \frac{1}{z_t^J (A_t^J)^{1-\alpha_J} (P_t^J R_t^J)^{\alpha_J} (W_t^J)^{1-\alpha_J} \left[ (1 - \alpha_J)^{1-\alpha_J} \alpha_J^{\alpha_J} \right]}$$

  2. Optimal production $Y_t^J$:

  $$M_t^J = Y_t^J \left[ \frac{1 - \gamma_J MC_t^{V,J}}{\gamma_J P_t^{ME}} \right]^{\gamma_J}$$
  
  $$V_t^J = Y_t^J \left[ \gamma_J \frac{P_t^{ME}}{1 - \gamma_J MC_t^{V,J}} \right]^{1-\gamma_J}$$
  
  $$MC_t^J = (MC_t^{V,J})^{\gamma_J} (P_t^{ME})^{1-\gamma_J} \left[ \frac{1}{(1 - \gamma_J)^{1-\gamma_J} \gamma_J^{\gamma_J}} \right]$$
B.1.4 Market Clearing

All markets clear:

\[ B_t = B_t^G \]
\[ I_t = I_t^X + I_t^N \]
\[ h_t^X = \Delta_t^{WX} h_t^{X,d} \]
\[ h_t^N = \Delta_t^{WN} h_t^{N,d} \]
\[ Y_t^X = \Delta_t^X \left( C_t^X + \tilde{I}_t^X + C_t^{X,*} \right) \]
\[ Y_t^M = \Delta_t^M \left( C_t^M + \tilde{I}_t^M + M_t^X + M_t^N \right) \]
\[ Y_t^N = \Delta_t^N \left( C_t^N + \tilde{I}_t^N + G_t^N \right) \]

Which correspond to the local bonds market, the investment market, labor markets and goods market. The \( \Delta \) variables are a measure of the dispersion in prices in the different markets.

The rest of the equations correspond to the policy and foreign equations described in the text.

B.2 Equilibrium Conditions

This sections describes the equilibrium conditions after the variables were redefined to make them stationary. The transformations made to the variables were: all lower case prices are the corresponding capital price divided by the CPI Index with the exception of \( p_t^{Co,*} \) and \( p_t^{M,*} \) which are divided by the foreign CPI price index and \( p_t^{J,*} = P_t^{J,*} / P_t^{J} \). All lower case real variables (consumption, investment, capital, government expenditure, production, imports, productivity, output, foreign demand) are the upper case divided by \( A_t^{-1} \) with the exception of \( y_t^{Co} = Y_t^{Co} / A_t^{-1} \). All inflation definitions are the corresponding price index divided by the price index in the previous period. And particular definitions are: \( \tilde{\xi}_t^{h,n} = \frac{\tilde{\xi}_t^{h,n}}{A_{t-1}}, \tilde{\mu}_t^I = \mu_t^I / P_t, \tilde{b}_t^J = B_t^J / (A_t^{-1} P_t), \tilde{\tilde{f}}_t^{I,J} = f_t^{I,J} / (A_t^{-1} P_t), \tilde{f}_t^{M,J} = f_t^{M,J} / A_t^{-1}, \tilde{\lambda}_t = P_t \lambda_t / A_t^{-1}, \tilde{w}_t^J = W_t^J / (A_t^{-1} P_t), \tilde{w}_t^{J,*} = W_t^{J,*} / W_t^J, mc_t^I = MC_t^I / P_t^I \) and \( mc_v^{J,*} = MC_v^{J,*} / P_t^J \) for \( J = \{ X, M, N \} \) or \( J = \{ X, N \} \) depending on the variable. In addition, new variables were defined as the real exchange rate, the trade balance, the gdp deflator among others.

There are 80 endogenous variables,

\[ \{ c_t, \tilde{\lambda}_t, h_t^X, \tilde{\mu}_t^{WX}, w_t^X, h_t^N, \mu_t^{WN}, w_t^N, R_t, \pi_t, R_t^*, \pi_t^*, C_t^X, \tilde{\mu}_t^I, p_t^I, \tilde{\mu}_t^X, p_t^X, R_t^X, p_t^{N}, p_t^N, R_t^N, p_t^I, i_t^X, i_t^N, k_t^X, k_t^N, f_t^{WX}, w_t^{X,*}, h_t^{X,d}, \pi_t^I, f_t^{WN}, w_t^{N,*}, h_t^{N,d}, \pi_t^N, p_t^{SAE}, T_t, T_t^*, T_t^X, T_t^N, M_t^X, M_t^N, \tilde{\xi}_t^N, \tilde{\xi}_t^X, \tilde{\xi}_t^M, i_t, mc_t^{M}, \tilde{\xi}^M, m_t, mt_{mt}, \tilde{\xi}_t^X, a_t^X, e_t^N, mc_t^{M}, mc_t^{V,N}, y_t^{X}, m_t^{ME}, m_t^X, m_t^N, mc_t^X, mc_t^N, f_t^{1,X}, p_t^{X,*}, f_t^{1,N}, p_t^{N,*}, gdp_t, \pi_t^{SAE}, \pi_t^{X,*}, \pi_t^{N,*}, \pi_t^{M,*}, R_t^{W}, \xi_{r_t}^{R}, \xi_{r_t}^{R2}, p_t^{Co,*}, y_t^{Co} \} \]

and 23 shocks:

\[ \{ \xi_t^C, y_t^C, \xi_t^{h,X}, \xi_t^{h,N}, \xi_t^N, \xi_t^X, \xi_t^{P,A}, \xi_t^{P,T}, \xi_t^{E}, \xi_t^{T}, \xi_t^{U}, \xi_t^{Z}, \xi_t^{Z,N}, \xi_t^{G}, \xi_t^{Y}, \xi_t^{E}, \xi_t^{Y}, \xi_t^{S}, \xi_t^{S}, \xi_t^{P}, \xi_t^{M,*}, R_t^{W}, \xi_{r_t}^{R}, \xi_{r_t}^{R2}, p_t^{Co,*}, y_t^{Co} \} \]
\[ \xi_t^\beta \left( c_t - \phi_C \frac{c_{t-1}}{a_{t-1}} \right)^{-\sigma} = \tilde{\lambda}_t \]  

(EC.1)

\[ \tilde{c}_{t, X}^h \left( h_t^X \right)^\rho = \mu_t^W \tilde{X}_t \]  

(EC.2)

\[ \tilde{c}_{t, N}^h \left( h_t^N \right)^\rho = \mu_t^{WN} \tilde{W}_t \]  

(EC.3)

\[ \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \tilde{\lambda}_{t+1} R_t \]  

(EC.4)

\[ \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \frac{\tilde{\lambda}_{t+1} R_t^{\pi_{t+1}}}{\pi_{t+1}} \]  

(EC.5)

\[ \tilde{\mu}_t^X \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \left\{ \tilde{\lambda}_{t+1} \tilde{\mu}_t^X R_t^{X} + \tilde{\mu}_t^X \tilde{\lambda}_t + (1 - \delta) \right\} \]  

(EC.6)

\[ \tilde{\mu}_t^N \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \left\{ \tilde{\lambda}_{t+1} \tilde{\mu}_t^N R_t^{N} + \tilde{\mu}_t^N \tilde{\lambda}_t + (1 - \delta) \right\} \]  

(EC.7)

\[ \tilde{\lambda}_t p_t^I = \tilde{\mu}_t^X \tilde{\lambda}_t \left\{ 1 - \frac{\phi_I}{2} \left( \frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 - \phi_I \left( \frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right) \right\} u_t + \]  

\[ \beta a_t^{-\sigma} E_t \tilde{\mu}_t^X \tilde{\lambda}_{t+1} \phi_I \left( \frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 u_{t+1} \]  

(EC.8)

\[ \tilde{\lambda}_t p_t^I = \tilde{\mu}_t^N \tilde{\lambda}_t \left\{ 1 - \frac{\phi_I}{2} \left( \frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 - \phi_I \left( \frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right) \right\} u_t + \]  

\[ \beta a_t^{-\sigma} E_t \tilde{\mu}_t^N \tilde{\lambda}_{t+1} \phi_I \left( \frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 u_{t+1} \]  

(EC.9)

\[ k_t^X = \left[ 1 - \frac{\phi_I}{2} \left( \frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 \right] u_t i_t^X + \left( 1 - \delta \right) \frac{k_{t-1}^X}{a_{t-1}} \]  

(EC.10)

\[ k_t^N = \left[ 1 - \frac{\phi_I}{2} \left( \frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 \right] u_t i_t^N + \left( 1 - \delta \right) \frac{k_{t-1}^N}{a_{t-1}} \]  

(EC.11)

\[ \hat{f}_{t}^{W, X} = \frac{\epsilon - 1}{\epsilon} \left( \frac{w_t^{X, *}}{w_t^{X, *}} \right)^{1-\epsilon} \tilde{\lambda}_t \tilde{X}_t \]  

\[ \theta_{W, X} a_t^{-\sigma} E_t \left( \frac{w_t^{X, *}}{w_{t+1}^{X, *}} \right)^{1-\epsilon} \]  

\[ \left( \frac{a_t}{a_t} \right)^{\pi_t + 1} \]  

\[ \left( \frac{\pi_t + 1}{a_t} \right) \]  

\[ \frac{w_t^{X, *}}{w_t^{X, *}} \hat{f}_{t+1}^{W, X} \]  

(EC.12)
\[ f_{t+1} \theta W N = \left( \frac{\epsilon W - 1}{\epsilon W} \right) (w_t^{N,X})^{1-\epsilon W} \tilde{f}_t + \theta W N a_t^{1-\sigma} \beta E_t \frac{w_t^{N,X}}{w_{t-1}^{N,X}} \left[ \frac{a_t \left( (\pi_t^N)^{\theta W X} \pi_t^{1-\theta W X} \bar{\pi}_W \bar{\pi}_N \pi_t \right)}{\pi_t} \right]^{1-\epsilon W} \frac{w_{t+1}^{N,X}}{w_{t+1}^{N,X}} \tilde{f}_{t+1} \theta W N \]

\[ \text{(EC.13)} \]

\[ f_{t+1} \theta X = \left( \frac{\epsilon W}{\mu_t} \right) ^{-\epsilon W} \mu_t^{X} \tilde{f}_t + \theta W X a_t^{1-\sigma} \beta E_t \frac{w_t^{X}}{w_{t-1}^{X}} \left[ \frac{a_t \left( (\pi_t^X)^{\theta W X} \pi_t^{1-\theta W X} \bar{\pi}_W \bar{\pi}_X \pi_t \right)}{\pi_t} \right]^{1-\epsilon W} \frac{w_{t+1}^{X}}{w_{t+1}^{X}} \tilde{f}_{t+1} \theta X \]

\[ \text{(EC.14)} \]

\[ f_{t+1} \theta N = \left( \frac{\epsilon W}{\mu_t} \right) ^{-\epsilon W} \mu_t^{N}\tilde{f}_t + \theta W N a_t^{1-\sigma} \beta E_t \frac{w_t^{N}}{w_{t-1}^{N}} \left[ \frac{a_t \left( (\pi_t^N)^{\theta W X} \pi_t^{1-\theta W X} \bar{\pi}_W \bar{\pi}_N \pi_t \right)}{\pi_t} \right]^{1-\epsilon W} \frac{w_{t+1}^{N}}{w_{t+1}^{N}} \tilde{f}_{t+1} \theta N \]

\[ \text{(EC.15)} \]

\[ 1 = \theta W X \left( \frac{w_{t-1}^{X}}{w_t^{X}} \right) ^{1-\epsilon W} \left( 1 - \theta W X \left( \frac{w_t^{X}}{w_{t-1}^{X}} \right) ^{1-\epsilon W} \right) \frac{\left( (\pi_t^X)^{\theta W X} \pi_t^{1-\theta W X} \bar{\pi}_W \bar{\pi}_X \pi_t \right)}{\pi_t} \]

\[ \text{(EC.16)} \]

\[ 1 = \theta W N \left( \frac{w_{t-1}^{N}}{w_t^{N}} \right) ^{1-\epsilon W} \left( 1 - \theta W N \left( \frac{w_t^{N}}{w_{t-1}^{N}} \right) ^{1-\epsilon W} \right) \frac{\left( (\pi_t^N)^{\theta W X} \pi_t^{1-\theta W X} \bar{\pi}_W \bar{\pi}_N \pi_t \right)}{\pi_t} \]

\[ \text{(EC.17)} \]

\[ c_t^N = \gamma \left( \frac{p_t^N}{p_t^{SAE}} \right) ^{-\theta} c_t \]

\[ \text{(EC.18)} \]

\[ c_t^T = (1 - \gamma) \left( \frac{p_t^T}{p_t^{SAE}} \right) ^{-\theta} c_t \]

\[ \text{(EC.19)} \]

\[ c_t^X = \gamma T \left( \frac{p_t^X}{p_t^T} \right) ^{-\theta T} c_t^T \]

\[ \text{(EC.20)} \]

\[ c_t^M = (1 - \gamma T) \left( \frac{p_t^M}{p_t^T} \right) ^{-\theta T} c_t^T \]

\[ \text{(EC.21)} \]

\[ 1 = \left( \frac{p_t^{SAE}}{p_t^T} \right) ^{1-\gamma AC-\gamma EC} \left( \frac{p_t^A}{p_t^T} \right) ^{\gamma AC} \left( \frac{p_t^E}{p_t^T} \right) ^{\gamma EC} \]

\[ \text{(EC.22)} \]

\[ 1 = (1 - \gamma) \left( \frac{p_t^T}{p_t^{SAE}} \right) ^{1-\theta} + \gamma \left( \frac{p_t^N}{p_t^{SAE}} \right) ^{1-\theta} \]

\[ \text{(EC.23)} \]
\[ 1 = (1 - \gamma_T) \left( \frac{p_t^M}{p_t^l} \right)^{1-\theta_T} + \gamma_T \left( \frac{p_t^X}{p_t^l} \right)^{1-\theta_T} \]  
(EC.24) 

\[ p_t^l = \left( \gamma_I (p_t^N)^{1-\theta_I} + (1 - \gamma_I) (p_t^{T,I})^{1-\theta_I} \right)^{\frac{1}{1-\theta_I}} \]  
(EC.25) 

\[ p_t^{T,I} = \left( \gamma_{T,I} (p_t^X)^{1-\theta_{T,I}} + (1 - \gamma_{T,I}) (p_t^M)^{1-\theta_{T,I}} \right)^{\frac{1}{1-\theta_{T,I}}} \]  
(EC.26) 

\[ \tilde{t}_t^N = \gamma_I \left( \frac{p_t^N}{p_t^l} \right)^{-\theta_I} \]  
(EC.27) 

\[ \tilde{t}_t^T = (1 - \gamma_I) \left( \frac{p_t^{T,I}}{p_t^l} \right)^{-\theta_I} \]  
(EC.28) 

\[ \tilde{t}_t^X = \gamma_{T,I} \left( \frac{p_t^X}{p_t^{T,I}} \right)^{-\theta_{T,I}} \]  
(EC.29) 

\[ \tilde{t}_t^M = (1 - \gamma_{T,I}) \left( \frac{p_t^M}{p_t^{T,I}} \right)^{-\theta_{T,I}} \]  
(EC.30) 

\[ m_{c_t}^M = \frac{p_t^{M,t}}{p_t^l} \]  
(EC.31) 

\[ y_t^M = m_t \]  
(EC.32) 

\[ h_t^{X,d} = \frac{v_t^X}{z_t^X(a_t^X)^{1-\alpha_X}} \left[ \frac{1 - \alpha_X}{\alpha_X} \frac{p_t^X}{w_t^X} R_t^X \right]^{\alpha_X} \]  
(EC.33) 

\[ k_t^{X,-1} = a_t-1 \frac{v_t^X}{z_t^X(a_t^X)^{1-\alpha_X}} \left[ \frac{\alpha_X}{1 - \alpha_X} \frac{w_t^X}{p_t^X R_t^X} \right]^{1-\alpha_X} \]  
(EC.34) 

\[ h_t^{N,d} = \frac{v_t^N}{z_t^N a_t^{1-\alpha_N}} \left[ \frac{1 - \alpha_N}{\alpha_N} \frac{p_t^N}{w_t^N R_t^N} \right]^{\alpha_N} \]  
(EC.35) 

\[ k_t^{N,-1} = a_t-1 \frac{v_t^N}{z_t^N a_t^{1-\alpha_N}} \left[ \frac{\alpha_N}{1 - \alpha_N} \frac{w_t^N}{p_t^N R_t^N} \right]^{1-\alpha_N} \]  
(EC.36) 

\[ m_{c_t}^{X,X} = \frac{1}{z_t^X(a_t^X)^{1-\alpha_X}} \left[ \frac{(p_t^X R_t^X)^{\alpha_X} (w_t^X)^{1-\alpha_X}}{p_t^X} \right] \frac{1}{1 - (1 - \alpha_X)^{1-\alpha_X} \alpha_X^{\alpha_X}} \]  
(EC.37) 

\[ m_{c_t}^{X,N} = \frac{1}{z_t^N a_t^{1-\alpha_N}} \left[ \frac{(p_t^N R_t^N)^{\alpha_N} (w_t^N)^{1-\alpha_N}}{p_t^N} \right] \frac{1}{1 - (1 - \alpha_N)^{1-\alpha_N} \alpha_N^{\alpha_N}} \]  
(EC.38) 

\[ v_t^X = y_t \left[ \frac{\gamma_X}{\gamma_X} \frac{p_t^{ME}}{m_{c_t}^{X,X}} \right]^{1-\gamma_X} \]  
(EC.39)
\[ m^X_t = y^X_t \left[ \frac{1 - \gamma_X m^{V,X}_t}{\gamma_X} \right]^{\gamma_X} \]  
(EC.40)

\[ v^N_t = y^N_t \left[ \frac{\gamma_N}{1 - \gamma_N m^{N}_t} \right]^{1-\gamma_N} \]  
(EC.41)

\[ m^N_t = y^N_t \left[ \frac{1 - \gamma_N m^{V,N}_t}{\gamma_N} \right]^{\gamma_N} \]  
(EC.42)

\[ m_{c}^X = (m_{c}^{V,X})^{\gamma_X} \left( \frac{p^{ME}_t}{p^X_t} \right)^{1-\gamma_X} \frac{1}{(1 - \gamma_X) \gamma_X} \]  
(EC.43)

\[ m_{c}^N = (m_{c}^{V,N})^{\gamma_N} \left( \frac{p^{ME}_t}{p^N_t} \right)^{1-\gamma_N} \frac{1}{(1 - \gamma_N) \gamma_N} \]  
(EC.44)

\[ a^X_t = \frac{a^X_{t-1}}{a_{t-1}} \tag{EC.45} \]

\[ p^{ME}_t = (p^M_t)^{1-\gamma_{EF}} (p^E_t)^{\gamma_{EF}} \]  
(EC.46)

\[ \bar{f}^{1,X}_t = \xi^{X} \frac{\epsilon^{X} - 1}{\epsilon^{X}} \left( p^{X}_{t} \right)^{1-\epsilon^{X}} y^{X}_{t} + \beta a^{1-\sigma^{X}} \theta^{X} E_t \left( \frac{p^{X}_{t}}{p^{X}_{t+1}} \right)^{1-\epsilon^{X}} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_{t}} \frac{\left( \pi^{X}_{t} \right)^{\epsilon^{X}}}{\pi^{1-\epsilon^{X}}_{t+1}} \frac{\xi^{1-\epsilon^{X}}}{\xi^{1-\epsilon^{X}}_{t+1}} \bar{f}^{1,X}_{t+1} \]  
(EC.47)

\[ \bar{f}^{1,M}_t = \xi^{M} \frac{\epsilon^{M} - 1}{\epsilon^{M}} \left( p^{M}_{t} \right)^{1-\epsilon^{M}} y^{M}_{t} + \beta a^{1-\sigma^{M \ast}} \theta^{M} E_t \left( \frac{p^{M}_{t}}{p^{M}_{t+1}} \right)^{1-\epsilon^{M}} \tilde{\lambda}_{t+1} = \frac{\left( \pi^{M}_{t} \right)^{\epsilon^{M}}}{\pi^{1-\epsilon^{M}}_{t+1}} \frac{\xi^{1-\epsilon^{M}}_{t+1}}{\xi^{1-\epsilon^{M}}_{t+1}} \bar{f}^{1,M}_{t+1} \]  
(EC.48)

\[ \bar{f}^{1,N}_t = \xi^{N} \frac{\epsilon^{N} - 1}{\epsilon^{N}} \left( p^{N}_{t} \right)^{1-\epsilon^{N}} y^{N}_{t} + \beta a^{1-\sigma^{N \ast}} \theta^{N} E_t \left( \frac{p^{N}_{t}}{p^{N}_{t+1}} \right)^{1-\epsilon^{N}} \tilde{\lambda}_{t+1} = \frac{\left( \pi^{N}_{t} \right)^{\epsilon^{N}}}{\pi^{1-\epsilon^{N}}_{t+1}} \frac{\xi^{1-\epsilon^{N}}_{t+1}}{\xi^{1-\epsilon^{N}}_{t+1}} \bar{f}^{1,N}_{t+1} \]  
(EC.49)
\[
\tilde{f}_t^{1,X} = \left( p_t^{X,*} \right)^{-\epsilon_X} m_{t+1}^{X} y_t^{X} + \\
\beta a_t^{1-\sigma} \theta_X E_t \left( \frac{p_t^{X,*}}{p_{t+1}^{X,*}} \right)^{-\epsilon_X} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\left( \pi_t^{X} \right)^{\sigma_X} \pi_t^{1-\epsilon_X}}{\pi_{t+1}^{-\epsilon_X}} \right) \tilde{f}_t^{1,X} \\
\frac{\pi_{t+1}^{X}}{\pi_{t+1}^{L+1}} \\
(\text{EC.50})
\]

\[
\tilde{f}_t^{1,M} = \left( p_t^{M,*} \right)^{-\epsilon_M} m_{t+1}^{M} y_t^{M} + \\
\beta a_t^{1-\sigma} \theta_M E_t \left( \frac{p_t^{M,*}}{p_{t+1}^{M,*}} \right)^{-\epsilon_M} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\left( \pi_t^{M} \right)^{\sigma_M} \pi_t^{1-\epsilon_M}}{\pi_{t+1}^{-\epsilon_M}} \right) \tilde{f}_t^{1,M} \\
\frac{\pi_{t+1}^{M}}{\pi_{t+1}^{L+1}} \\
(\text{EC.51})
\]

\[
\tilde{f}_t^{1,N} = \left( p_t^{N,*} \right)^{-\epsilon_N} m_{t+1}^{N} y_t^{N} + \\
\beta a_t^{1-\sigma} \theta_N E_t \left( \frac{p_t^{N,*}}{p_{t+1}^{N,*}} \right)^{-\epsilon_N} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\left( \pi_t^{N} \right)^{\sigma_N} \pi_t^{1-\epsilon_N}}{\pi_{t+1}^{-\epsilon_N}} \right) \tilde{f}_t^{1,N} \\
\frac{\pi_{t+1}^{N}}{\pi_{t+1}^{L+1}} \\
(\text{EC.52})
\]

\[
\pi_t^{X} = \frac{p_t^{X}}{p_{t-1}^{X}} \\
(\text{EC.53})
\]

\[
\pi_t^{M} = \frac{p_t^{M}}{p_{t-1}^{M}} \\
(\text{EC.54})
\]

\[
\pi_t^{N} = \frac{p_t^{N}}{p_{t-1}^{N}} \\
(\text{EC.55})
\]

\[
\pi_t^{SAE} = \frac{p_t^{SAE}}{p_{t-1}^{SAE}} \\
(\text{EC.56})
\]

\[
1 = (1 - \theta_X) \left( \tilde{p}_t^{*,X} \right)^{1-\epsilon_X} + \theta_X \left( \left( \pi_t^{X} \right)^{\sigma_X} \pi_t^{1-\epsilon_X} \right) \tilde{f}_t^{1,X} \left( \frac{1}{\pi_t^{X}} \right)^{1-\epsilon_X} \\
(\text{EC.57})
\]

\[
1 = (1 - \theta_M) \left( \tilde{p}_t^{*,M} \right)^{1-\epsilon_M} + \theta_M \left( \left( \pi_t^{M} \right)^{\sigma_M} \pi_t^{1-\epsilon_M} \right) \tilde{f}_t^{1,M} \left( \frac{1}{\pi_t^{M}} \right)^{1-\epsilon_M} \\
(\text{EC.58})
\]

\[
1 = (1 - \theta_N) \left( \tilde{p}_t^{*,N} \right)^{1-\epsilon_N} + \theta_N \left( \left( \pi_t^{N} \right)^{\sigma_N} \pi_t^{1-\epsilon_N} \right) \tilde{f}_t^{1,N} \left( \frac{1}{\pi_t^{N}} \right)^{1-\epsilon_N} \\
(\text{EC.59})
\]

\[
\left( \frac{R_t}{R} \right)^{\theta_R} \left( \frac{R_{t-1}}{R} \right)^{\theta_R} \left( \left( \frac{\pi_t^{SAE}}{\pi} \right)^{\alpha_{SAE}} \pi_t^{1-\alpha_{SAE}} \right)^{\alpha_{SAE}} \left( \frac{gdp_t a_t-1}{gdp_{t-1}} \right)^{\alpha Y} \tilde{f}_t^{1,Y} \\
(\text{EC.60})
\]
\[ \xi_{t}^{X,*} = \left( \frac{P_{t}^{X}}{\text{rer}_{t}} \right)^{-\epsilon^{*}} y_{t}^{*} \xi_{t}^{X,*} \]  

(EC.61)

\[ \frac{\text{rer}_{t}}{\text{rer}_{t-1}} = \pi_{t}^{S} \pi_{t}^{S} \pi_{t} \]  

(EC.62)

\[ p_{m,t} = \text{rer}_{t} p_{m,t} \]  

(EC.63)

\[ R_{t}^{*} = R_{t}^{W} \exp \left\{ \phi_{B} \left( \bar{b} - \frac{h_{t}^{*} \text{rer}_{t}}{p_{t}^{X} g_{dp_{t}}} \right) \right\} \xi_{t}^{R} \xi_{t}^{R2} \]  

(EC.64)

\[ i_{t} = i_{t}^{X} + i_{t}^{N} \]  

(EC.65)

\[ h_{t}^{X} = \Delta_{t}^{W} h_{t}^{X,d} \]  

(EC.66)

\[ h_{t}^{N} = \Delta_{t}^{W} h_{t}^{N,d} \]  

(EC.67)

\[ y_{t}^{N} = \Delta_{t}^{N} (c_{t}^{N} + g_{t} + \bar{i}_{t}^{N}) \]  

(EC.68)

\[ y_{t}^{X} = \Delta_{t}^{X} (c_{t}^{X} + \bar{i}_{t}^{X} + c_{t}^{X,*}) \]  

(EC.69)

\[ y_{t}^{M} = \Delta_{t}^{M} (c_{t}^{M} + \bar{i}_{t}^{M} + m_{t}^{X} + m_{t}^{N}) \]  

(EC.70)

\[ \Delta_{t}^{W X} = (1 - \theta_{WX}) \left( w_{t}^{X,*} \right)^{-\epsilon_{W}} + \theta_{WX} \left( \frac{w_{t-1}^{X} a}{a_{t-1}} \left( \frac{w_{t}^{X}}{\pi_{t}} \right)^{\phi_{W} \pi_{t-1}^{X} \pi_{t-1}^{W X} \bar{\pi}_{1}^{1 - \phi_{W X}}} \right)^{-\epsilon_{W}} \Delta_{t-1}^{W X} \]  

(EC.71)

\[ \Delta_{t}^{W N} = (1 - \theta_{WN}) \left( w_{t}^{N,*} \right)^{-\epsilon_{W}} + \theta_{WN} \left( \frac{w_{t-1}^{N} a}{a_{t-1}} \left( \frac{w_{t}^{N}}{\pi_{t}} \right)^{\phi_{WN} \pi_{t-1}^{N} \pi_{t-1}^{WN} \bar{\pi}_{1}^{1 - \phi_{WN}}} \right)^{-\epsilon_{W}} \Delta_{t-1}^{W N} \]  

(EC.72)

\[ \Delta_{t}^{X} = (1 - \theta_{X}) \left( p_{t}^{X,*} \right)^{-\epsilon_{X}} + \theta_{X} \left( \frac{\pi_{X}^{X} \phi_{X} \pi_{t-1}^{X} \bar{\pi}_{1}^{1 - \phi_{X}}} {\pi_{t}^{X}} \right)^{-\epsilon_{X}} \Delta_{t-1}^{X} \]  

(EC.73)

\[ \Delta_{t}^{M} = (1 - \theta_{M}) \left( p_{t}^{M,*} \right)^{-\epsilon_{M}} + \theta_{M} \left( \frac{\pi_{M}^{M} \phi_{M} \pi_{t-1}^{M} \bar{\pi}_{1}^{1 - \phi_{M}}} {\pi_{t}^{M}} \right)^{-\epsilon_{M}} \Delta_{t-1}^{M} \]  

(EC.74)

\[ \Delta_{t}^{N} = (1 - \theta_{N}) \left( p_{t}^{N,*} \right)^{-\epsilon_{N}} + \theta_{N} \left( \frac{\pi_{N}^{N} \phi_{N} \pi_{t-1}^{N} \bar{\pi}_{1}^{1 - \phi_{N}}} {\pi_{t}^{N}} \right)^{-\epsilon_{N}} \Delta_{t-1}^{N} \]  

(EC.75)

\[ tb_{t} = \text{rer}_{t} \frac{C_{t} a_{t-1}^{C_{t}}}{a_{t-1}} + p_{t}^{X} c_{t}^{X,*} - p_{m,t} m_{t} \]  

(EC.76)
\[
\begin{align*}
\alpha^C_t &= \left( \frac{\alpha^C_{t-1}}{\alpha^C_{t-1}} \right)^{1-\Gamma^C_t} \frac{\alpha^C_t}{\alpha^C_t} \quad \text{(EC.77)}
\end{align*}
\]

\[
\begin{align*}
\text{rer}_t b^*_t &= t_t + \frac{\text{rer}_t}{\pi^*_t} R^*_t a^*_{t-1} - (1 - \vartheta) \text{rer}_t p^*_{t} \frac{\alpha^*_{t-1}}{\alpha^*_{t-1}} - m_t \\
gdp_t &= c_t + g_t + i_t + c^*_t + y_t \frac{\alpha^*_{t-1}}{\alpha^*_{t-1}} - m_t \\
p^*_t \text{gdp}_t &= c_t + p^*_t g_t + p^*_t i_t + t_t \\
\end{align*}
\]

The equations for the exogenous processes are described in the text.

B.3 Steady State

The given endogenous are: \( \{R, h^X, h^N, p^X/p^I, p^M/p^I, s^C = \text{rer}, p^C, s^M = p_M y^M/(p^Y \text{gdp}), s^g = p^N g/(p^Y \text{gdp})\} \) and the exogenous variables that are calculated endogenously are: \( \{\beta, \tilde{\xi}^h, \tilde{\xi}^N, z^X, g, y^*, \pi^*, \gamma, \bar{b}\} \).

By (EC.64) (assuming that the part inside the bracket is zero):
\[
R^* = R^W \xi^R
\]

By (EC.45)
\[
a^X = a^X^{2X-1}
\]

By (EC.77)
\[
a^C = a^C^{2C-1}
\]

By (EC.60) and (EC.56) (assuming \( \epsilon^m = 1 \)):
\[
\pi^{SAE} = \pi = \bar{\pi}
\]

By (EC.4):
\[
\beta = \frac{a^\sigma \pi}{R}
\]

By (EC.5):
\[
\pi^S = \frac{a^\sigma \pi}{R^* \beta}
\]

By (EC.62):
\[
\pi^* = \frac{\pi}{\pi^S}
\]

By (EC.63)-(EC.65):
\[
\pi^X = \pi^M = \pi^N = \pi
\]

By (EC.57)-(EC.59):
\[
p^X, s = p^M, s = p^N, s = 1
\]

By (EC.16)-(EC.17):
\[
w^X, s = w^N, s = 1
\]
By (EC.71)-(EC.75):

\[ \Delta W X = \Delta W N = \Delta X = \Delta M = \Delta N = 1 \]

By (EC.47)-(EC.52)

\[ m_c X = \frac{\epsilon_X - 1}{\epsilon_X} \]
\[ m_c M = \frac{\epsilon_M - 1}{\epsilon_M} \]
\[ m_c N = \frac{\epsilon_N - 1}{\epsilon_N} \]

By (EC.12)-(EC.15)

\[ \mu W X = \mu W N = \frac{\epsilon_W - 1}{\epsilon_W} \]

By (EC.66)-(EC.67)

\[ h_{X,d} = h_X \]
\[ h_{N,d} = h_N \]

From the relative prices \( p^X / p^I \) and \( p^M / p^I \), we get using (EC.24)-(EC.26) the relative prices:

\[ \frac{p^{T,I}}{p^I} = \left( \gamma_{T,I} \left( \frac{p^X}{p^I} \right)^{1-\sigma_{T,I}} + (1 - \gamma_{T,I}) \left( \frac{p^M}{p^I} \right)^{1-\sigma_{T,I}} \right) \]
\[ \frac{p^N}{p^I} = \left( 1 - (1 - \gamma_I) \left( \frac{p^{T,I}}{p^I} \right)^{1-\sigma_I} \right)^{1-\sigma_I} \]
\[ \frac{p^P}{p^I} = \left[ (1 - \gamma_T) \left( \frac{p^M}{p^I} \right)^{1-\sigma_T} + \gamma_T \left( \frac{p^X}{p^I} \right)^{1-\sigma_T} \right]^{1-\sigma_T} \]

From (EC.8)-(EC.9):

\[ \frac{\tilde{\mu} X}{p^I} = \frac{\tilde{\mu} N}{p^I} = 1/u \]

By (EC.6)-(EC.7):

\[ R^X = \frac{(\tilde{\mu} X / p^I)(1 - \beta a^{-\sigma}(1 - \delta))}{\beta a^{-\sigma}(p^X / p^I)} \]
\[ R^N = \frac{(\tilde{\mu} N / p^I)(1 - \beta a^{-\sigma}(1 - \delta))}{\beta a^{-\sigma}(p^N / p^I)} \]

By (EC.31):

\[ \frac{p_m}{p^I} = m c M (p^M / p^I) \]

By (EC.63):

\[ \frac{rer}{p^I} = \frac{p_m}{p^I} \frac{p^I}{p_m} \]
It is further assumed that \( p^A = p^E = p^T \), and so, we also have \( p^A/p^I \) and \( p^E/p^I \). By (EC.46):

\[
\frac{p^M}{p^I} = \left( \frac{p^E}{p^I} \right)^{1-\gamma_E} \left( \frac{p^T}{p^I} \right)^{\gamma_E}
\]

By (EC.43)-(EC.44):

\[
mc^{V,X} = \left( \frac{mc^X (p^X/p^I)^{1-\gamma_X} (1 - \gamma_X) \gamma_X^{\gamma_X}}{p^ME/p^I} \right)^{1-\gamma_X}
\]

\[
mc^{V,N} = \left( \frac{mc^N (p^N/p^I)^{1-\gamma_N} (1 - \gamma_N) \gamma_N^{\gamma_N}}{p^ME/p^I} \right)^{1-\gamma_N}
\]

By (EC.38):

\[
\frac{w^N}{p^I} = \left( \frac{mc^{V,N} z^N a^{1-\alpha_N} (p^N/p^I) (1 - \alpha_N) \gamma_N^{\alpha_N}}{(p^N/p^I)^{\alpha_N}} \right)^{1-\alpha_N}
\]

By (EC.3):

\[
\frac{\tilde{\xi}^{h,N}}{p^I} = \frac{\mu^W N}{(h^N)^{\varphi}} \frac{w^N}{p^I}
\]

Assuming that \( \tilde{\xi}^{h,X} = \tilde{\xi}^{h,N} \), we also have \( \tilde{\xi}^{h,X}/p^I \) and with (EC.2):

\[
\frac{w^X}{p^I} = \left( \frac{(\tilde{\xi}^{h,X}/p^I)(h^X)^{\varphi}}{\mu^W X} \right)
\]

By (EC.37):

\[
z^X = \frac{((p^X/p^I)^{\alpha_X} (w^X/p^I)^{1-\alpha_X})}{mc^{V,X} (a_X)^{1-\alpha_X} (p^X/p^I) (1 - \alpha_X)^{1-\alpha_X} \gamma_X^{\alpha_X}}
\]

By (EC.33) and (EC.35):

\[
v^X = h^{X,d} z^X (a_X)^{1-\alpha_X} \left[ \frac{\alpha_X}{1 - \alpha_X} \frac{w^X/p^I}{(p^X/p^I)^{\alpha_X}} \right]^{\alpha_X}
\]

\[
v^N = h^{N,d} z^N a^{1-\alpha_N} \left[ \frac{\alpha_N}{1 - \alpha_N} \frac{w^N/p^I}{(p^N/p^I)^{\alpha_N}} \right]^{\alpha_N}
\]

By (EC.34) and (EC.36):

\[
k^X = a \frac{v^X}{z^X (a_X)^{1-\alpha_X}} \left[ \frac{\alpha_X}{1 - \alpha_X} \frac{w^X/p^I}{(p^X/p^I)^{\alpha_X}} \right]^{1-\alpha_X}
\]

\[
k^N = a \frac{v^N}{z^N a^{1-\alpha_N}} \left[ \frac{\alpha_N}{1 - \alpha_N} \frac{w^N/p^I}{(p^N/p^I)^{\alpha_N}} \right]^{1-\alpha_N}
\]

By (EC.39) and (EC.41):

\[
y^X = v^X \left[ \frac{\gamma_X}{1 - \gamma_X} \frac{p^M/E/p^I}{mc^{V,X}} \frac{1}{p^X/p^I} \right]^{-(1-\gamma_X)}
\]
\[ y^N = y^N \left[ \frac{\gamma_N}{1 - \gamma_N} \frac{p^{ME}/p^I}{mc^{V,N}/p^N/p^I} \right]^{(1 - \gamma_N)} \]

By (EC.40) and (EC.42):

\[ m^X = y^X \left[ \frac{1 - \gamma_X}{\gamma_X} \frac{mc^{V,X}/p^{ME}/p^I}{p^X/p^I} \right]^\gamma_X \]
\[ m^N = y^N \left[ \frac{1 - \gamma_N}{\gamma_N} \frac{mc^{V,N}/p^{ME}/p^I}{p^N/p^I} \right]^\gamma_N \]

By (EC.47) and (EC.49):

\[ \tilde{f}^{1,X} = \frac{\epsilon_X - 1}{\epsilon_X} \frac{y^X}{(1 - \beta a^{1-\sigma} \theta_X)} \]
\[ \tilde{f}^{1,N} = \frac{\epsilon_N - 1}{\epsilon_N} \frac{y^N}{(1 - \beta a^{1-\sigma} \theta_N)} \]

By (EC.10)-(EC.11):

\[ i^X = \frac{k^X}{u} \left( 1 - \frac{1 - \delta}{a} \right) \]
\[ i^N = \frac{k^N}{u} \left( 1 - \frac{1 - \delta}{a} \right) \]

By (EC.65):

\[ i = i^X + i^N \]

By (EC.27)-(EC.30):

\[ \tilde{r}^N = \gamma_I \left( \frac{p^N}{p^I} \right)^{-\hat{\theta}_I} \tilde{i} \]
\[ \tilde{r}^T = (1 - \gamma_I) \left( \frac{p^{T,I}}{p^I} \right)^{-\hat{\theta}_I} \tilde{i} \]
\[ \tilde{r}^X = \gamma_{T,I} \left( \frac{p^X/p^I}{p^{T,I}/p^I} \right)^{-\hat{\theta}_{T,I}} \tilde{i}^T \]
\[ \tilde{r}^M = (1 - \gamma_{T,I}) \left( \frac{p^M/p^I}{p^{T,I}/p^I} \right)^{-\hat{\theta}_{T,I}} \tilde{i}^T \]

When replacing equations (EC.68)-(EC.70) into equation (EC.80) (and using the identities of expenditures), one gets an alternative sum for nominal gdp:

\[ p^Y gdp = p^X y^X + rer p^C a \theta a^C + p^N y^N + p^M y^M - p^M (m^X + m^N) - p_m m \]

which can also be written in terms of prices relative to investment:

\[ \frac{p^Y}{p^I} gdp = \frac{p^X}{p^I} y^X + \frac{rer}{p^I} p^C a \theta a^C + \frac{p^N}{p^I} y^N + \frac{p^M}{p^I} y^M - \frac{p^M}{p^I} (m^X + m^N) - \frac{p_m m}{p^I} \]
And using $s^{Co}, s^M$:

$$\frac{p^Y}{p^T} gdp = \frac{\frac{p^X}{p^T} y^X + \frac{p^N}{p^T} y^N - \frac{p^M}{p^T} (m^X + m^N)}{1 - s^{Co} - s^M (\frac{(p^M - p_m)/p^I}{p_m/p^I})}$$

With this, we can get:

$$y^{Co} = \frac{s^{Co}(p^Y/p^I) gdp}{(rer/p^I) p^{Co,*} a^{Co}}$$

$$y^M = \frac{s^M(p^Y/p^I) gdp}{p_m/p^I}$$

$$g = \frac{s^g(p^Y/p^I) gdp}{p^N/p^I}$$

By (EC.51):

$$\bar{\mu}^{1,M} = \frac{\epsilon_M - 1}{\epsilon_M} \frac{y^M}{(1 - \beta a^{1-\sigma} \theta_M)}$$

By (EC.32):

$$m = y^M$$

By (EC.68):

$$c^N = y^N - g - \tilde{r}^N$$

By (EC.70):

$$c^M = y^M - \tilde{r}^M - m^X - m^N$$

By (EC.21):

$$c^T = \frac{c^M}{1 - \gamma_T} \left( \frac{p^M/p^I}{p^T/p^I} \right)^{\theta_T}$$

By (EC.20):

$$c^X = \gamma_T \left( \frac{p^X/p^I}{p^T/p^I} \right)^{-\theta_T} c^T$$

By (EC.18)-(EC.19):

$$\gamma = \frac{(p^N/p^I)^\theta c^N}{(p^T/p^I)^\theta c^I + (p^N/p^I)^\theta c^N}$$

By (EC.22)-(EC.23):

$$\frac{p_{SAE}}{p^I} = \left[ (1 - \gamma) \left( \frac{p^T}{p^I} \right)^{1-\theta} + \gamma \left( \frac{p^N}{p^I} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$p^I = \left[ \left( \frac{p_{SAE}}{p^I} \right)^{1-\gamma_A} \left( \frac{\gamma_A}{p^T} \right)^{\gamma_A} \left( \frac{\gamma_A}{p^I} \right)^{\gamma_A} \right]^{-1}$$

Now, we get all prices by multiplying the price relative to investment by $p^I$: 

$$\{ p^X, p^M, p^N, p^T, p^I, p_{SAE}, p_{ME}, rer, u^X, u^N, \tilde{\mu}^X, \tilde{\mu}^N, p_m, \tilde{r}_b, N \}$$
By (EC.18):
\[ c = \frac{1}{\gamma} (p^N)^\sigma c^N \]
(also check equation \( c = c^T (p^T)^\sigma / (1 - \gamma) \))

By (EC.69):
\[ c^{X,*} = y^X - \tilde{y}^X - c^X \]

By (EC.61):
\[ y^* = \frac{c^{X,*}}{\xi^{X,*}} \left( \frac{p^X}{rer} \right)^e^{*} \]

By (EC.79):
\[ gdp = c + g + i + c^{X,*} + \frac{y_{Co}a^{Co}}{a} - m \]
\[ p^Y = \frac{p^Y gdp}{gdp} \]

By (EC.76):
\[ tb = rer \ p^{Co,*} y_{Co}a^{Co} - p^X c^{X,*} - p_m m \]

By (EC.78):
\[ b^* = \frac{tb - (1 - \vartheta) rer \ p^{Co,*} y_{Co}a^{Co}}{rer \left( 1 - \frac{R^*}{\pi^* a} \right)} \]

By (EC.64) (part that was assumed zero):
\[ \tilde{b} = \frac{b^* rer}{p^Y gdp} \]

By (EC.1):
\[ \tilde{\lambda} = \xi^\beta c^{-\alpha} \left( 1 - \frac{\phi a}{a} \right)^{-\sigma} \]

By (EC.14)-(EC.15):
\[ \tilde{f}^{1,WX} = \frac{\mu^{WX} \tilde{h}_{X,d}}{1 - \theta_{WX} a^{1-\sigma} \beta} \]
\[ \tilde{f}^{1,WN} = \frac{\mu^{WN} \tilde{h}_{N,d}}{1 - \theta_{WN} a^{1-\sigma} \beta} \]
## C Parameterization

Table 3: Calibrated

<table>
<thead>
<tr>
<th>Para.</th>
<th>Descrip.</th>
<th>Value</th>
<th>Source</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>Risk Aversion</td>
<td>1</td>
<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Inv. Frish elast.</td>
<td>1</td>
<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Share $C^N$ in $C^{NFE}$</td>
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<td>I-O Matrix, average 08-13</td>
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<tr>
<td>$\gamma_T$</td>
<td>Share $C^X$ in $C^T$</td>
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<td>I-O Matrix, average 08-13</td>
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<td>Share $I^N$ in $I$</td>
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<td>Share $I^X$ in $I^T$</td>
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<td>$\gamma_{EC}$</td>
<td>Share $C^E$ in $C$</td>
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<tr>
<td>$\pi$</td>
<td>Inflation (annual)</td>
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<tr>
<td>$a$</td>
<td>Long-run growth (annual)</td>
<td>1.016</td>
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<tr>
<td>$R^W$</td>
<td>World Interest Rate (annual)</td>
<td>1.045</td>
<td>Average 01-15</td>
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<tr>
<td>$R$</td>
<td>Monetary Policy Rate (annual)</td>
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<td>$\delta$</td>
<td>Capital depreciation</td>
<td>0.01</td>
<td>Medina and Soto (2007)</td>
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<tr>
<td>$\epsilon$</td>
<td>Elast. of Subst. Varieties</td>
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<td>Medina and Soto (2007)</td>
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### Table 4: Estimated Parameters

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<th>Para.</th>
<th>Description</th>
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<th>St.D.</th>
<th>Mode</th>
<th>St.D.</th>
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<td>φ_{I}</td>
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<td>Calvo $W^X$</td>
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<td>η</td>
<td>Sust. $IT, IN$</td>
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<td>Adj. Trend $X$</td>
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<td>Γ_{Co}</td>
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**Policy Rule**

<p>| | | | | |</p>
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<td>α_{y}</td>
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<td>Elast. Ext. Dem.</td>
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Note: Prior distributions: β Beta, $N⁺$ Normal truncated for positive values, IG Inverse Gamma, U Uniform. The standard deviation of the posterior is approximated by the inverse Hessian evaluated at the posterior mode.
Table 5: Estimated Parameters, Coefficients Dynamics of Exogenous Processes

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<th>Dynamic of Driving Forces</th>
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<th>Posterior</th>
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<td>$\rho_{z}$</td>
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<td>$\Gamma_{M*}$</td>
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<td>$\Gamma_{Co*}$</td>
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<td>$\rho_{*}$</td>
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<td>$\rho_{M*}$</td>
<td>$\mathcal{U}$</td>
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Table 6: Estimated Parameters, Standard Deviations Exogenous shocks

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Facultad de Ciencias Económicas
Departamento de Investigación “Francisco Valsecchi”

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